Chapter 1

1-1

$$1\ \text{yr} = 365\ \frac{\text{days}}{\text{yr}} \times 24\ \frac{\text{hrs}}{\text{day}} = 8760\ \text{hrs}$$

Average Wasted Power $P_{avg} = \dfrac{100 \times 10^9\ \text{kWh/yr}}{8760\ \text{hr/yr}} = 11.41 \times 10^3\ \text{MW}$

(a)

$\therefore \sim 11\frac{1}{2}$ 1000-MW generating plants running continuously provide this power.

(b) Annual Savings $= 0.10\ \dfrac{\$}{\text{kWh}} \times 100 \times 10^9\ \dfrac{\text{kWh}}{\text{yr}}$

$= 10$ Billion $/yr

1-5

For the ease of solving this problem, let us assume that the system operates for 100 hrs and draws 1 kW while delivering 100% flow rate.

The percentage reduction in the power consumption is the same as the percentage reduction in the energy consumption.

The following Table shows the energy consumption using each of the three methods:

hrs of Operation	Drive	Outlet	Inlet
20 hrs @ 100%	1 × 20 kWh	1 × 20 kWh	1 × 20 kWh
20 hrs @ 80%	0.5 × 20	0.92 × 20	0.81 × 20
30 hrs @ 60%	0.3 × 30	0.87 × 30	0.7 × 30
10 hrs @ 30%	0.1 × 10	0.72 × 10	0.65 × 10
Total Energy Consumption	$E_{Drive} = 40$ kWh	$E_{outlet} = 71.7$ kWh	$E_{inlet} = 63.7$ kWh

Using an adjustable speed drive,

(a) % reduction over outlet damper $= \dfrac{E_{outlet} - E_{Drive}}{E_{outlet}}$

$$= \frac{71.7 - 40}{71.7} \times 100 \simeq 44\%$$

(b) % reduction over inlet vanes $= \dfrac{E_{inlet} - E_{Drive}}{E_{inlet}}$

$$= \frac{63.7 - 40}{63.7} \times 100 \simeq 37\%$$

1-5

$$\lambda = k \frac{\omega_m r}{V_{wind}}$$

$$V_{wind, rated} = 12 \text{ m/s}$$

$$V_{wind, cut\ in} = 4 \text{ m/s}$$

For maximum value of C_p, $\omega_m = 20$ rpm at the rated wind.

$\therefore$ ω_m at the cut in wind speed is

$$20 \times \frac{4}{12} = 6.66 \text{ rpm}$$

$\therefore$ the blade rotational speed should vary between 6.66 rpm to 20 rpm between the cut-in and the rated wind speeds, in order to keep C_p at its maximum value.

Chapter 2

2-1

$$J \frac{d\omega_m}{dt} = T$$

$T = 5 \text{ Nm}$

$$\frac{d\omega_m}{dt} = \frac{1800-0}{3} \frac{rpm}{s} = \frac{1800}{3} \times \frac{1}{60} \times 2\pi \ \text{rad}/s^2$$

$$= 62.83 \ \frac{\text{rad}}{s^2}$$

$$J = \frac{T}{\frac{d\omega_m}{dt}} = \frac{5 \text{ Nm}}{62.83 \ \frac{\text{rad}}{s^2}} \cong 0.08 \ kg.m^2$$

2-2

Modifying Eq. 2-16 for a hollow cylinder

$$\left(\rho \int_{r_2}^{r_1} r^3 dr \int_0^{2\pi} d\theta \int_0^{l} dl\right) \frac{d\omega_m}{dt} = T$$

or

$$\underbrace{\left(\rho \left.\frac{r^4}{4}\right|_{r_2}^{r_1} 2\pi l\right)}_{J_{cyl,hollow}} \frac{d\omega_m}{dt} = T$$

$$\therefore \quad J_{cyl,hollow} = \frac{\pi}{2} \rho l \left(r_1^4 - r_2^4\right)$$

$$= \frac{\pi}{2} \times 7.85 \times 10^3 \times 0.18 \times (0.06^4 - 0.04^4)$$

$$= 0.0174 \ kg.m^2$$

2-3

$$50\ \frac{km}{hr} = \frac{50\times10^3}{3600}\ \frac{m}{s} = 13.89\ \frac{m}{s}$$

and, $10\ \frac{km}{hr} = 2.78\ \frac{m}{s}$

Eq 2-9 $\quad W_M = \frac{1}{2} M u^2$

$$\Delta W_{M,\,recovered} = \frac{1}{2} M u^2 \Big|_{u=13.89\frac{m}{s}} - \frac{1}{2} M u^2 \Big|_{u=2.78\frac{m}{s}}$$

$$= \frac{1}{2} \times 1500 \left(13.89^2 - 2.78^2\right)$$

$$= 138.9\ \ kJ$$

2-4

$$u = r\,\omega_m$$

$$\therefore\ \omega_m = \frac{u}{r} = \frac{1\ m/s}{0.09\ m} = 11.11\ rad/s$$

$$\frac{d\omega_m}{dt} = \frac{11.11-0}{4} = 2.777\ \frac{rad}{s^2}$$ (linear acceleration due to constant torque)

Eq. 2-38

$$T_{em} = \left(J_m + r^2 M\right)\frac{d\omega_m}{dt}$$

$$= \left(0.01 + 0.09^2 \times 1\right) \times 2.777$$

$$= 0.05\ Nm$$

2-5 The load-speed profile is given — that is, $\frac{du}{dt}$ is given.

$$u = r\omega_m \Rightarrow \omega_m = \frac{u}{r}$$

and $\frac{d\omega_m}{dt} = \frac{du}{dt}\frac{1}{r}$

Substituting for $\frac{d\omega_m}{dt}$ in Eq. 2-38 ($f_L=0$)

$$T_{em} = \left(\frac{J_m}{r} + rM\right)\frac{du}{dt}$$

$$= r\left[M + \frac{J_m}{r^2}\right]\frac{du}{dt}$$

To minimize Tem for a given $\frac{du}{dt}$, we will take a partial derivative of Tem (with respect to r) and set it to zero —

$$\frac{\partial T_{em}}{\partial r} = \left\{\left(M + \frac{J_m}{r^2}\right) + r\left[-2\frac{J_m}{r^3}\right]\right\}\frac{du}{dt} = 0$$

$$\therefore \quad M + \frac{J_m}{r^2} - 2\frac{J_m}{r^2} = 0$$

or, $M = \frac{J_m}{r^2} \Rightarrow r = \sqrt{\frac{J_M}{M}}$

$$= \sqrt{\frac{40\times10^{-3}\times10^{-4}}{0.02}}$$

$$= 0.0141\text{m}$$

$$= 1.41 \text{ cm}$$

We can take a second derivative with respect to r to confirm that what we have is the minimum (<u>not</u> the maximum).

2-6

The length of the arc between two teeth is the same in two gears. Therefore, for the motor-side gear

$$\text{arc length} \times n_M = 2\pi\, r_M$$

Similiarly,

$$\text{arc length} \times n_L = 2\pi\, r_L$$

$$\therefore \quad \frac{n_M}{n_L} = \frac{r_M}{r_L} = \frac{\omega_L}{\omega_M} \quad \text{(using Eq. 2-41)}$$

$$= \frac{1}{3}$$

$$\therefore \quad \omega_M = 3\,\omega_L$$

during $0 \le t \le 1$ s

$$\frac{d\omega_L}{dt} = 100 \text{ rad/s}^2 \quad \therefore \frac{d\omega_M}{dt} = 300 \text{ rad/s}^2$$

From Eq. 2-43

$$T_{em} = \left[J_M + \left(\frac{\omega_L}{\omega_M}\right)^2 J_L\right]\frac{d\omega_M}{dt} = \left[1.2 + \left(\frac{1}{3}\right)^2 10.0\right] \times 300$$

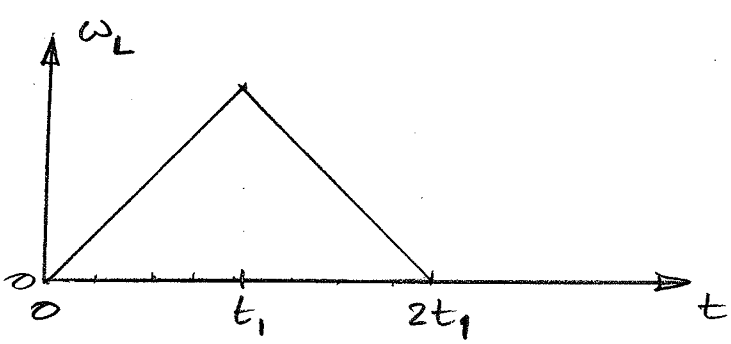
$$= 693.33 \text{ Nm}$$

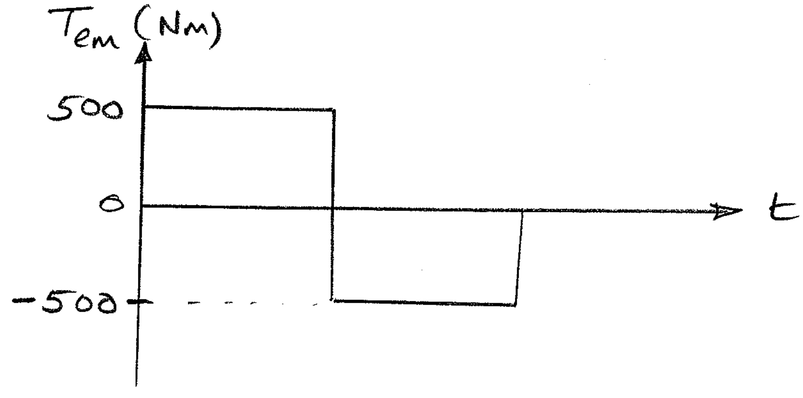
$1 \le t \le 2$ $\quad T_{em} = 0$ $\left(\text{Since } \frac{d\omega_L}{dt} = 0\right)$

$2 \le t \le 3$ $\quad T_{em} = -693.33$ Nm

$3 \le t \le 4$ $\quad T_{em} = 0$

2-7





During $0 \le t < t_1$, from Eq. 2-43

$$T_{em} = 500 = \left[1.2 + \left(\tfrac{1}{3}\right)^2 10.0\right] \frac{d\omega_m}{dt}$$

$$\therefore \frac{d\omega_M}{dt} = \frac{500}{\left[1.2 + \left(\frac{1}{3}\right)^2 \times 10\right]} = 216.35 \text{ rad/s}^2$$

$$\Rightarrow \frac{d\omega_L}{dt} = \frac{1}{3}\frac{d\omega_M}{dt} = 72.11 \frac{\text{rad}}{s^2}$$

$\therefore$ during $0 \le t \le t_1$,

$$\omega_L = 72.11\, t \text{ rad/s}$$

Due to the symmetry of the speed profile during acceleration and deceleration, the load over the interval t_1 rotates by an angle $= \frac{30^\circ}{2} = 15^\circ$.

$$\therefore \theta_L(t_1) = \int_0^{t_1} \omega_L(\tau)\cdot d\tau = \int_0^{t_1} 72.11\,\tau \cdot d\tau = \frac{72.11}{2} t_1^2$$

$$= \frac{15 \times \pi}{180} \text{ rad}$$

(assuming that $\theta_L(0) = 0$)

$$\therefore \quad t_1 = \sqrt{\frac{2 \times 15 \times \pi}{72.11 \times 180}} = 85.2 \text{ ms}$$

Therefore, the time required to rotate the load by an angle of 30° is

$$2t_1 = 0.17 \text{ s.}$$

2-8

Vehicle maximum speed $= 150 \frac{km}{hr} = \frac{150 \times 10^3}{3600}$

$$\therefore \quad u_L = 41.667 \frac{m}{s}$$

$\therefore$ wheel's maximum rotational speed

$$\omega_L = \frac{u_L}{r_L} = \frac{41.667}{(0.6/2)} = 138.89 \frac{rad}{s}$$

wheel radius

$$= \frac{138.89}{2\pi} \times 60$$

$$= 1326.29 \text{ rpm}$$

This should occur at the maximum motor speed of 5000 rpm. Therefore, using the relationship derived in Problem 2-6,

(a) $$\frac{n_M}{n_L} = \frac{\omega_L}{\omega_M} = \frac{1326.29}{5000} = 0.265$$

(b) From Table 2-1, $f_L = 930.9$ N

$$\therefore \quad f_{L,\text{ per wheel}} = \frac{f_L}{4} = \frac{930.9}{4} \text{ N}$$

$$T_{L,\text{ per wheel}} = r_L \, f_{L,\text{ wheel}} = 0.3 \times \frac{930.9}{4} = 69.82 \text{ Nm}$$

$$\therefore \quad T_{M,\text{ per motor}} = \frac{\omega_L}{\omega_M} T_{L,\text{ per wheel}} = 0.265 \times 69.82$$

$$= 18.5 \text{ Nm}$$

2-9

From Eq. 2-45a in Fig. 2-15,

$$\left(\frac{r_1}{r_2}\right)_{opt.} = \sqrt{\frac{J_M}{J_L}} = \sqrt{\frac{40}{60}} = 0.816$$

2-10

In the Lead-screw system, a rotation of θ_m ($= 2\pi$ rad) corresponds to a linear movement x_L ($= s$ m).

Therefore, $\dfrac{x_L}{\theta_m} = \dfrac{s}{2\pi} = n$ (1)

At a speed u_L, the stored kinetic energy by the mass $(M_T + M_W)$ is $\frac{1}{2}(M_T + M_W)\, u_L^2$. In terms of an equivalent rotating mass of inertia J_L', the kinetic energy would be $\frac{1}{2} J_L' \omega_m^2$. Equating these two energy expressions,

$$\frac{1}{2} J_L' \omega_m^2 = \frac{1}{2}(M_T + M_W)\, u_L^2$$

or $$J_L' = (M_T + M_W)\left(\frac{u_L}{\omega_m}\right)^2 \qquad (2)$$

Differentiating both sides of Eq.(1) with respect to time,

$$\frac{u_L}{\omega_m} = n \qquad (3)$$

Substituting Eq. (3) into Eq. (2),

$$J_L' = (M_T + M_W)\, n^2 \qquad (4)$$

The electromagnetic torque T_L' required from the motor to overcome F_L can be calculated by equating the work done –

$$T_L'\, \theta_m = F_L\, x_L$$

$$\therefore \quad T_L' = \frac{x_L}{\theta_m} F_L = n\, F_L \quad \text{(using Eq. 1)} \qquad (5)$$

Therefore, the total electromagnetic torque required from the motor is

$$T_{em} = \underbrace{(J_m + J_s + J_L')\frac{d\omega_m}{dt}}_{\text{inertia component}} + n F_L \qquad (6)$$

but, $$\frac{d\omega_m}{dt} = \frac{1}{n}\frac{du_L}{dt} = \frac{\dot{u}_L}{n} \quad \text{(using Eq. 3)} \qquad (7)$$

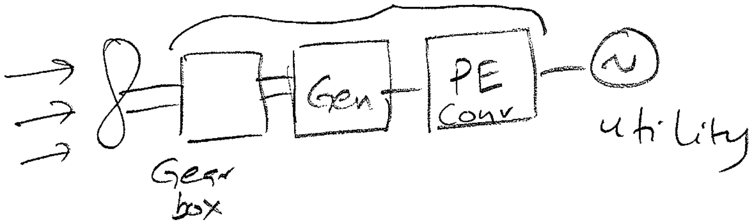
Substituting Eq. (7) into Eq. (6)

$$T_{em} = \frac{\dot{u}_L}{n}(J_M + J_s + J_L') + n F_L$$

$$= \frac{\dot{u}_L}{n}\left[J_m + J_s + \underbrace{n^2 (M_T + M_W)}_{\text{using Eq. (4)}}\right] + n F_L$$

Problem 2-11

$\eta = 90\%$



$C_p = 0.48, \quad A_r = 4000 \, m^2$

$\rho = 1.2 \, \frac{kg}{m^3}, \quad V_w = 13 \, \frac{m}{s}$

$$P_{shaft} = C_p \left(\tfrac{1}{2} \rho A_r V_w^3 \right)$$

$$= 0.48 \times \tfrac{1}{2} \times 1.2 \times 4000 \times 13^3$$

$$= 2.53 \text{ MW}$$

Problem 2-12

$\eta = 100\%$ assume

at the wind speed of 13 m/s and 22 rpm,

$P_{shaft} = 1.5$ MW $= P_{\text{delivered to the grid}}$ (assuming $\eta = 100\%$)

$$\therefore \; T_{shaft} = \frac{P}{\omega_m} = \frac{1.5 \times 10^6 \, W}{\frac{22}{60} \times 2\pi \, \frac{rad}{s}} = 0.651 \times 10^6 \, Nm$$

assuming that T_{shaft} remains constant during the disturbance $\Delta t = 2s$ when $P = 0$

$$J_{eq} \frac{\Delta \omega}{\Delta t} = T_{shaft}$$

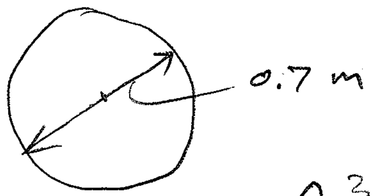
$$\therefore \; \Delta \omega = \frac{0.651 \times \cancel{10^6} \times (2)}{3.4 \times \cancel{10^6}} = 0.383 \, \frac{rad}{s}$$

$$= 0.383 \times \frac{60}{2\pi} = 3.66 \text{ rpm}$$

Problem 2-13

$M = 2000\,kg$ $\quad$ $1\text{ mile} = 1.6093\text{ km}$

0 to 60 mph linearly in 10 s

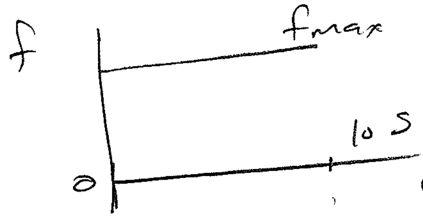


$r = 0.35\,m$

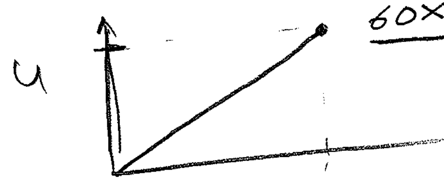
$$f_{max} = M\frac{du}{dt}$$

$$= \frac{2000}{10} \times \frac{60 \times 1.6093 \times 10^3}{60 \times 60}$$

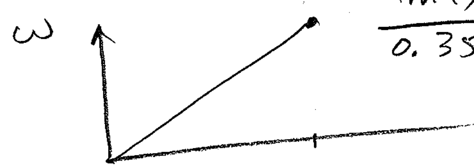
$$= 5.364 \times 10^3\,N$$



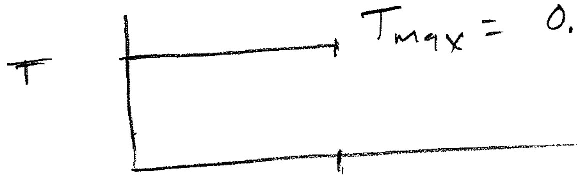
$$u_{max} = \frac{60 \times 1.6093 \times 10^3}{60 \times 60}\ \frac{m}{s}$$

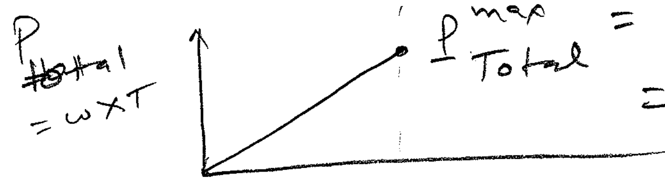


$$\frac{u_{max}}{0.35}\ \frac{rad}{s}$$



$$T_{max} = 0.35 \times f_{max}$$





$$P_{Total}^{max} = \frac{u_{max}}{0.35} \times 0.35 \times f_{max}$$

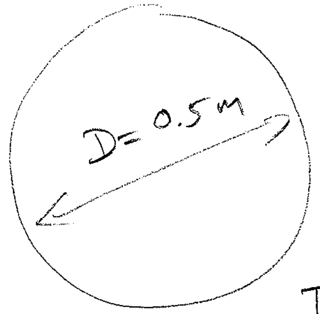
$$= u_{max} f_{max}$$

$$\therefore P_{wheel}^{max} = \frac{u_{max} f_{max}}{4} = \frac{(60 \times 1.6093 \times 10^3) \times 5.364 \times 10^3}{4 \times 60 \times 60}$$

$$= 35.97\,kW$$

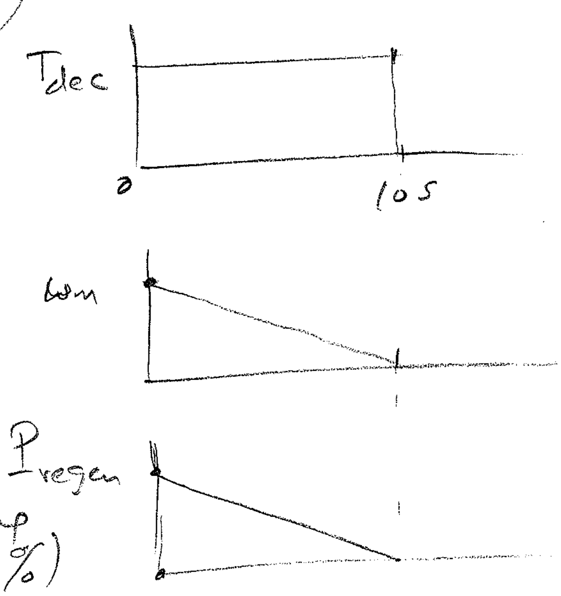
Problem 2-14

$$M = 1000 \text{ kg}$$



$$acc = \frac{\Delta u}{\Delta t} = \frac{20 \text{ m/s}}{10}$$

For each wheel:



(assuming $\eta = 100\%$)

$$f_{dec} = M \frac{du}{dt} = 1000 \times \frac{20}{10} = 2000 \text{ N}$$

$$T_{dec} = r\, f_{dec} = 0.25 \times 2000 = 500 \text{ Nm}$$

$$\omega_{m,max} = \frac{u_{max}}{r} = \frac{20}{0.25} = 80 \text{ rad/s}$$

$$P_{regen}^{wheel} = \frac{1}{4}\left[T_{dec} \times \omega_{m,max}\right]$$

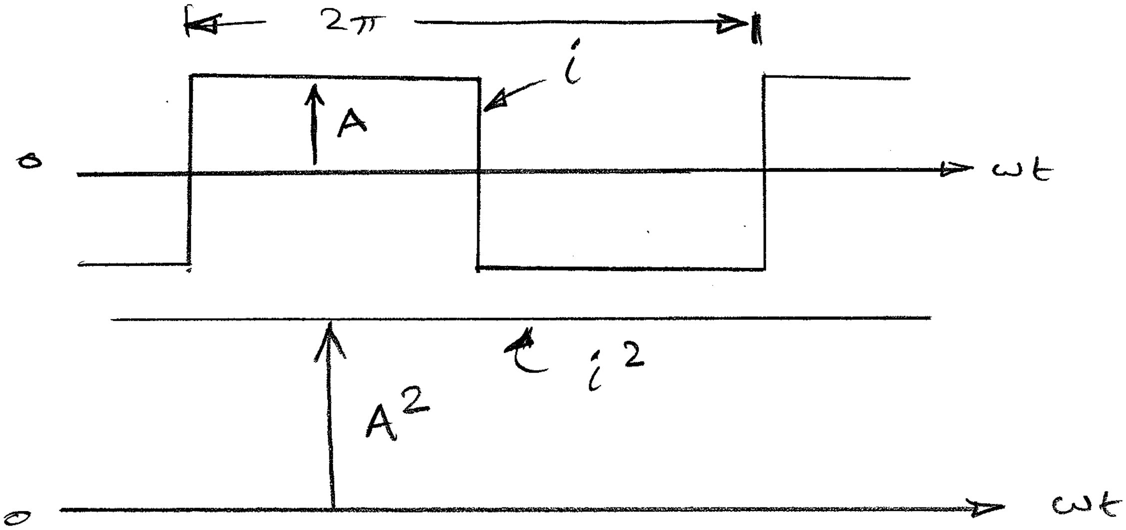
$$= \frac{1}{4} \times \frac{500 \times 80}{1000} \text{ kW}$$

$$= \underline{10 \text{ kW}}$$

Chapter 3

3-1

(a)



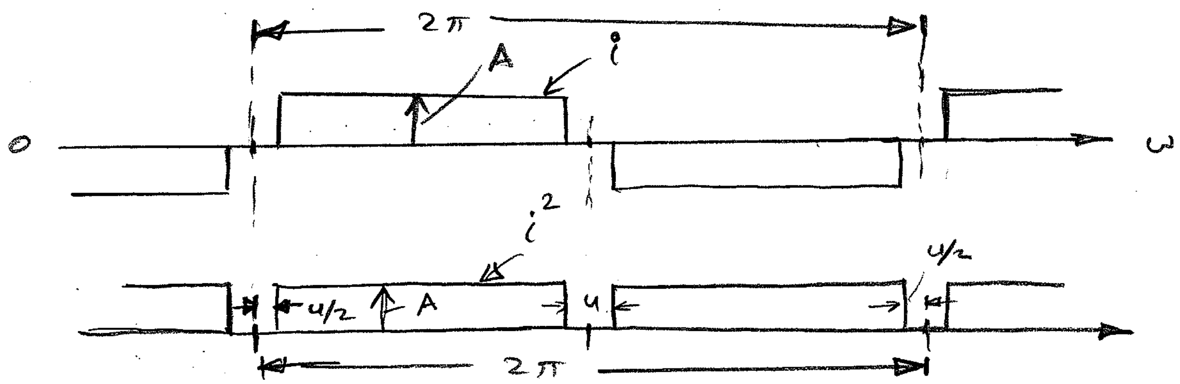
$$\therefore \quad I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \cdot dt}$$

[Note: T = period of repetition
$\omega = 2\pi f$
$f = \frac{1}{T}$ $\quad \therefore \omega T = 2\pi$]

$$= \sqrt{\frac{1}{\omega T}\int_0^{2\pi} i^2 \cdot d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \cdot d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi} A^2 \cdot 2\pi} = A \quad \text{Amps}$$

(b)



$$\therefore \quad I_{rms} = \sqrt{\frac{A^2(2\pi - 2u)}{2\pi}} = A\sqrt{1 - \frac{u}{\pi}} \quad \text{Amps}$$

3-2 Repeat of Problem 3-1.

3-3 (a) $v_1 = \sqrt{2} \times 120 \cos(\omega t - 30^\circ)$ V

$= 169.7 \cos(\omega t - 30^\circ)$ V

$\therefore \bar{V}_1 = 169.7 \angle -30^\circ$ V

(b) Similarly,

$\bar{V}_2 = 169.7 \angle +30^\circ$ V

3-4 Solve the differential equation Eq. 3-3

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = \hat{V}\cos\omega t \quad (1)$$

Differentiating with respect to time,

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i = -\omega\hat{V}\sin\omega t \quad (2)$$

In steady state, the form of $i(t)$ will be

$$i(t) = \hat{I}\cos(\omega t + \phi_i) \quad (3)$$

where $\hat{I}$ and ϕ_i need to be determined.

The above expression for $i(t)$ can also be written as

$$i(t) = A\cos\omega t + B\cos\omega t \quad (4)$$

where A and B are coefficients to be determined.

Using the expression for $i(t)$ in Eq. 4 into Eq. 2,

$$-\omega RA \sin\omega t + B\omega R\cos\omega t - L\omega^2 A\cos\omega t - L\omega^2 B\sin\omega t + \frac{A}{C}\cos\omega t + \frac{B}{C}\sin\omega t = -\omega\hat{V}\sin\omega t$$

Collecting $\sin\omega t$ and $\cos\omega t$ terms on both sides,

$$-\omega RA - L\omega^2 B + \frac{B}{C} = -\omega\hat{V} \quad \text{---} \quad (5)$$

and,

$$B\omega R - L\omega^2 A + \frac{A}{C} = 0 \quad \text{---} \quad (6)$$

From Eq. 6,

$$B = -\frac{1}{\omega R}\left[\frac{1}{C} - L\omega^2\right]A \quad \text{---} \quad (7)$$

where from Eq. 5,

$$(-\omega R)A + \left(\frac{1}{C} - L\omega^2\right)B = -\omega\hat{V} \quad \text{---} \quad (8)$$

Substituting for B from Eq. 7 into Eq. 8

$$\left[-\left(\frac{1}{C} - L\omega^2\right)^2\frac{1}{\omega R} - \omega R\right]A = -\omega\hat{V}$$

$$\therefore\ A = \frac{\omega\hat{V}}{\dfrac{\left(\frac{1}{C} - L\omega^2\right)^2}{\omega R} + \omega R} \quad \text{---} \quad (9)$$

In this problem, $\hat{V} = \sqrt{2} \times 120 = 169.7V$

$R = 1.3\Omega$ $L = 0.02$ and $C = 100\mu F$.

$\omega = 2\pi \times 60 = 377$ rad/s

Substituting these values into Eq. 9,

$$A = 0.609 \qquad (10)$$

Substituting the value of A from Eq. 10 into Eq. 7

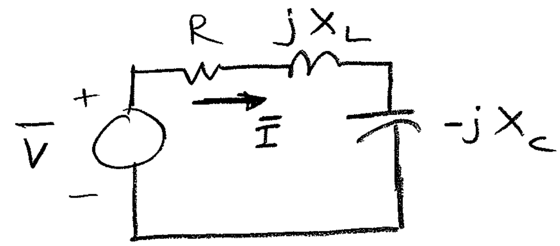
$$B = -8.894$$

$\therefore$ From Eq. 4,

$$i(t) = 0.609 \cos\omega t - 8.894 \sin \omega t \qquad (11)$$

We can reduce this to the form in Eq. 3 but this process is long enough already to convince us that it is quite tedius and the phasor-domain analysis shown in Problem 3-5 is much preferred.

3.5



$\bar{V} = 169.7 \angle 0^\circ$ V

$R = 1.3\,\Omega$

$X_L = 20\text{mH} \times 377 = 7.54\,\Omega$

$X_C = \dfrac{1}{377 \times 100\mu} = 26.526\,\Omega$

$$\bar{I} = \frac{\bar{V}}{R + jX_L - jX_C} = 8.92 \angle 86.08 \text{ A}$$

$$\therefore \; i(t) = 8.92 \cos(\omega t + 86.08^\circ) \text{ A}$$

In order to compare results with that in Problem 3-4, the expression for $i(t)$ can be expanded as

$$i(t) = 8.92\left[\cos\omega t \cdot \cos(86.08^\circ) - \sin\omega t \cdot \sin(86.08^\circ)\right]$$

$$= 0.609 \cos\omega t - 8.899 \sin\omega t \ \text{A}$$

which checks out with Eq. 11 of Problem 3-4.

3-6

$$\bar{V} = 90\angle 30^\circ \text{ V} \Rightarrow \bar{I} = 5\angle 15^\circ \text{ A}$$

Since it is a linear circuit, with

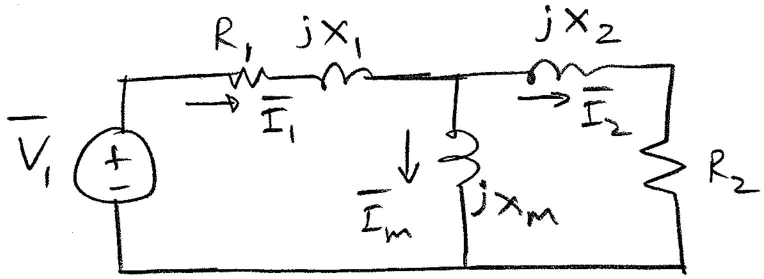
$$\bar{V} = 120\angle 0^\circ$$

$$\Rightarrow \bar{I} = 5\angle 15^\circ \ \frac{120\angle 0^\circ \leftarrow \text{new value}}{90\angle 30^\circ \leftarrow \text{old value}}$$

$$= \frac{5 \times 120}{90}\angle 15^\circ - 30^\circ$$

$$= 6.667\angle -15^\circ \ \text{A}$$

3-7



$\bar{V}_1 = \sqrt{2} \times 120 \angle 0°$
$R_1 = 0.3\,\Omega$
$X_1 = 0.5\,\Omega$
$X_m = j15\,\Omega$
$X_2 = 0.2\,\Omega$
$R_2 = 7\,\Omega$

At the terminals, from Example 3-2

$$P = VI\cos\phi = 120 \times \frac{25}{\sqrt{2}} \cos(29°) = 1855.3\,W$$

$$Q = VI\sin\phi = 120 \times \frac{25}{\sqrt{2}} \sin(29°) = 1028.4\ VAR$$

$\bar{I}_1 = 25\angle -29°\ A$

$$\therefore \bar{I}_m = \bar{I}_1 \frac{R_2 + jX_2}{(R_2 + jX_2) + jX_m} = 10.46 \angle -92.65°\ A$$

$$\bar{I}_2 = \bar{I}_1 - \bar{I}_m = 22.412 \angle -4.3°\ A$$

$$\therefore I_1 = \frac{25}{\sqrt{2}} = 17.678\,A, \quad I_2 = \frac{22.412}{\sqrt{2}} = 15.848\,A$$

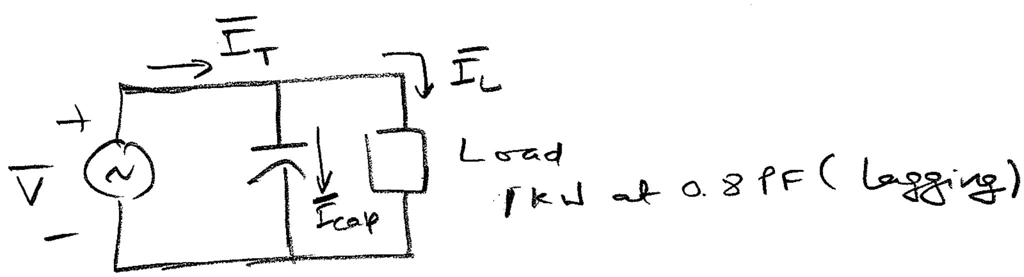
and $I_m = \frac{10.46}{\sqrt{2}} = 7.398\,A$

$$\therefore P_{R_1} = R_1 I_1^2 = 0.3 \times 17.678^2 = 93.754\,W$$
$$P_{R_2} = R_2 I_2^2 = 7 \times 15.848^2 = 1758.114\,W$$
$$\therefore \Sigma P = P_{R_1} + P_{R_2} = 1851.9\,W$$

$$Q_{X_1} = X_1 I_1^2 = 0.5 \times 17.678^2 = 156.256\ VAR$$
$$Q_{X_2} = X_2 I_2^2 = 0.2 \times 15.848^2 = 50.232\ VAR$$
$$Q_{X_m} = X_m I_m^2 = 15.0 \times 7.398^2 = 820.956\ VAR$$
$$\therefore \Sigma Q = Q_{X_1} + Q_{X_2} + Q_{X_3} = 1027.4\ VAR$$

Note that ΣP and ΣQ are equal to P and Q at the terminals (slightly different due to numerical roundoffs).

3-8



$$P_L = 1\text{ kW}, \quad pf = 0.8 \text{ (lagging)}$$

$$\therefore \quad VI_L = \frac{P}{PF} = \frac{1000}{0.8} = 1250 \text{ VA}$$

$$\therefore \quad Q_L = \sqrt{(VI_L)^2 - P_L^2}$$
$$= \sqrt{1250^2 - 1000^2} = 750 \text{ VAR}$$

After the power-factor correction,

$$VI_T = \frac{P_L}{0.95} = \frac{1000}{0.95} = 1052.63 \text{ VA}$$

$$\therefore \quad Q_T = \sqrt{(VI_T)^2 - P_L^2} = 328.68 \text{VAR}$$

$$Q_T = Q_{cap} + Q_L$$

$$\therefore \quad Q_{cap} = Q_T - Q_L$$
$$= 328.68 - 750$$
$$= -421.3 \text{ VAR}$$

$$|Q_{cap}| = V\, I_{cap} = V \frac{V}{1/\omega C} = V^2 \omega C$$

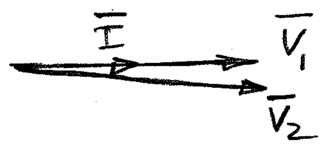
$$\therefore \quad C = \frac{|Q_{cap}|}{\omega V^2} = \frac{421.3}{377 \times 120^2}$$
$$\simeq 77.6\ \mu F$$

3-9 We work with rms values instead of peak values to represent phasor amplitudes.

$$\bar{V}_2 = \bar{V}_1 - jX_L\bar{I}$$

(a) $\bar{I} = 10\angle 0^\circ A$ $\therefore \bar{V}_2 = 120\angle 0^\circ - j0.5\times 10\angle 0^\circ$

$$= 120.104\angle -2.39^\circ\ V$$



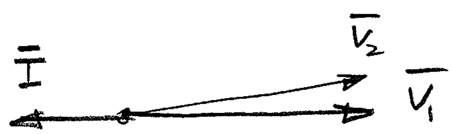
$$S_1 = \bar{V}_1\bar{I}^* = 120\angle 0^\circ \times 10\angle 0^\circ = \underbrace{1200}_{P_1} + j\underbrace{0}_{Q_1}$$

$\therefore P_1 = 1,200\ W,\quad Q_1 = 0\ VAR$

(b) $\bar{I} = 10\angle 180^\circ$

$\therefore \bar{V}_2 = 120\angle 0^\circ - j0.5(-10)$

$$= 120\angle 0^\circ + j5.0 = 120.104\angle +2.386^\circ\ V$$



$$S_1 = \bar{V}_1\bar{I}^* = 120\angle 0^\circ \times 10\angle -180^\circ = \underbrace{-1200}_{P_1} + j\underbrace{0}_{Q_1}$$

$\therefore P_1 = -1200\ W$

$Q_1 = 0$

(c) $\bar{I} = 10\angle 90^\circ$ A

$\therefore \quad \bar{V}_2 = 120\angle 0^\circ - j\,0.5 \times 10\angle 90^\circ$

$= 120\angle 0^\circ + 5.0 = 125\angle 0^\circ$ V

$S_1 = \bar{V}_1 \bar{I}^* = 120\angle 0^\circ \times 10\angle -90^\circ$

$= 1200\, e^{-j90^\circ}$

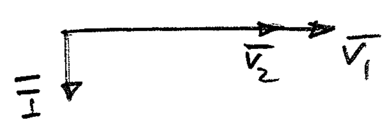
$= \underbrace{0}_{P_1} + j\underbrace{(-1200)}_{Q_1}$

$\therefore \quad P_1 = 0, \quad Q_1 = -1200$ VARS

(d) $\bar{I} = 10\angle -90^\circ$

$\therefore \quad \bar{V}_2 = 120\angle 0^\circ - j\,0.5 \times 10\angle -90^\circ$

$= 120\angle 0^\circ - 5.0 = 115\angle 0^\circ$ V



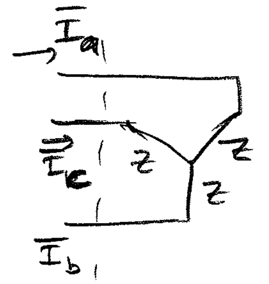
$S_1 = \bar{V}\,\bar{I}^* = 120\angle 0^\circ \times 10\angle 90^\circ$

$= 1200\, e^{j90^\circ}$

$= \underbrace{0}_{P_1} + j\,\underbrace{1200}_{Q_1}$

$\therefore \quad P_1 = 0, \quad Q_1 = +1200$ VARS

3-10



$V_{ph} = 120\,V$ (rms)

$P_{3\phi} = 10\ kW$ (at a pf $= 0.9$ lagging)

because it is given that the load is inductive.

$$P_{1-\phi} = \frac{P_{3-\phi}}{3} = \frac{10}{3}\ kw \text{ at } 0.85\,pf$$

$$V I \cos\phi = P_{1-\phi}$$

$$\therefore\ I = \frac{10000/3}{120 \times 0.85} = 32.68\ A$$

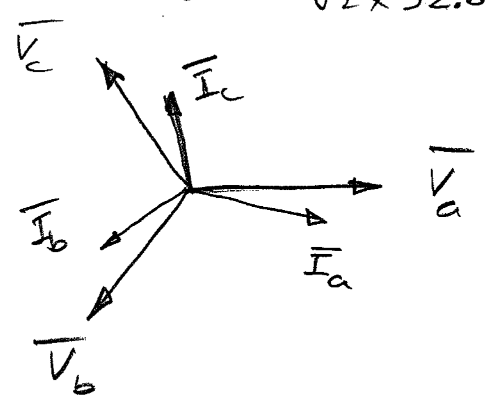
$$\cos\phi = 0.85 \quad \therefore\ \phi = 31.788^\circ$$

Assuming that $\bar{V} = \sqrt{2} \times 120 \angle 0^\circ\ V$ is the reference phasor,

note: due to lagging pf

$$\bar{I} = \sqrt{2} \times 32.68 \angle -31.788^\circ\ A$$

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{\sqrt{2} \times 120 \angle 0^\circ}{\sqrt{2} \times 32.68 \angle -31.788^\circ} = 3.67 \angle 31.788^\circ\ \Omega$$



3-11

$$\overline{V}_a = \sqrt{2} \times 100 \angle 30^\circ$$

$$v_a(t) = \sqrt{2} \times 100 \cos(\omega t + 30^\circ)\ V$$

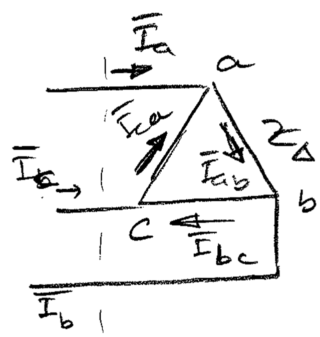
$$v_b(t) = \sqrt{2} \times 100 \cos(\omega t - 90^\circ)\ V$$

$$v_c(t) = \sqrt{2} \times 100 \cos(\omega t - 210^\circ)\ V$$

$$\overline{V}_{ab} = \overline{V}_a - \overline{V}_b = \sqrt{3}\ \overline{V}_a \angle 30^\circ = \sqrt{3} \times \sqrt{2} \times 100 \angle 60^\circ\ V$$

$$\therefore\ v_{ab}(t) = \sqrt{3} \times \sqrt{2} \times 100 \cos(\omega t + 60^\circ)\ V$$

3-12



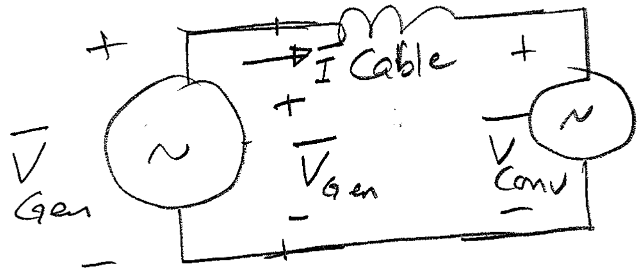
$$Z_\Delta = 3Z_y = 3 \times 3.67 \angle 31.788^\circ$$

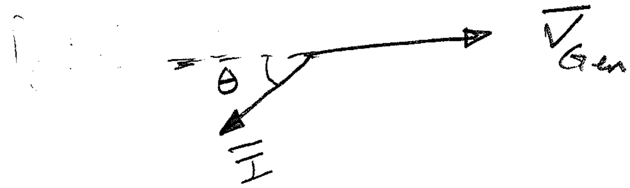
$$= 11.01 \angle 31.788^\circ\ \Omega$$

The line currents $\overline{I}_a$, $\overline{I}_b$ and $\overline{I}_c$ would be the same as in Problem 3-10.

3-13

Gen: 690V, 2.3 MW, pf = 0.85 lagging





$\theta = \cos^{-1}(0.85) = 31.79°$

$\therefore \quad \frac{Q}{P} = \tan\theta \quad \therefore \quad Q = 2.3(\tan 31.79)$ MVAR

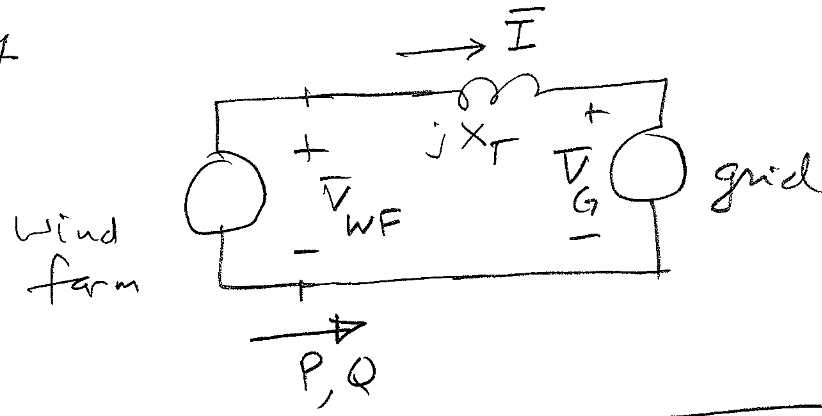
$= 1.425$ MVAR

$\sqrt{3}\, V_{LL} I \cos\theta = P$

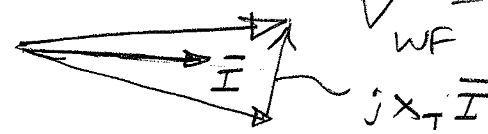
$\therefore \quad I = \frac{2.3 \times 10^6}{\sqrt{3} \times 690 \times 0.85}$

$\simeq 2264$ A

3-14



$\bar{V}_{WF} = 1\angle 0^\circ$ pu



$$S = P + jQ = 1 + j0.1 = \bar{V}_{WF}\,\bar{I}^*$$

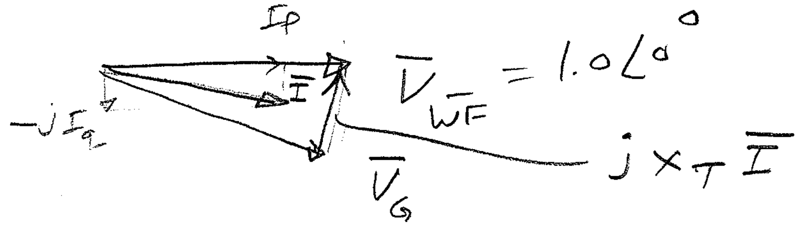
$$\therefore \bar{I} = 1 - j\,0.1 = 1.005\angle -5.71^\circ$$

$$\bar{V}_G = \bar{V}_{WF} - jX_T\bar{I} = 1.0\angle 0^\circ - j0.2\,(1 - j\,0.1)$$

$$= 1.0 - j0.2 - 0.02 = 0.98 - j0.2$$

$$= 1.0\angle 11.53^\circ$$

Prob 3-15



$\bar{V}_{WF} = 1.0\angle 0^\circ$ $\quad \bar{I} = I_p - jI_q$ $\quad I_q$ = positive number

$jX_T = j0.2$

$P + jQ = \bar{V}_{WF}\,\bar{I}^* = 1.0\,(I_p + jI_q)$

Since $P = 1$, $I_p = 1$

$$\bar{V}_G = \bar{V}_{WF} - j0.2\,\bar{I}$$
$$= 1.0 - j0.2(I_p - jI_q)$$
$$= 1.0 - j0.2 - 0.2\,I_q$$
$$= (1 - 0.2\,I_q) - j0.2$$

$$V_G = 1 = \sqrt{(1-0.2\,I_q)^2 + 0.04}$$

or, $(1-0.2\,I_q)^2 + 0.04 = 1$

or $(1-0.2\,I_q)^2 = 0.96$

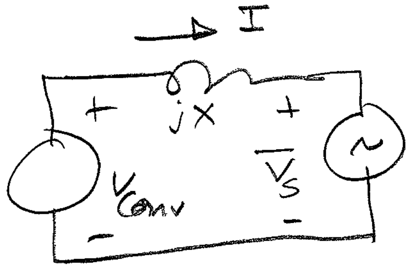
$\therefore$ $1 - 0.2\,I_q = 0.98 \Rightarrow I_q = 0.1$

$$S = P + jQ = \bar{V}_{WF}\,\bar{I}^* = 1 \times (1 + j0.1)$$
$$= 1 + j0.1$$

$\therefore$ $Q = 0.1$ pu supplied by wind farm

$\bar{V}_G = (1 - 0.2\times 0.1) - j0.2 = 0.98 - j0.2 = 1\angle -11.53^\circ$ pu

3-16



$jX = j\,0.05$ pu

$\bar{V}_{conv} = \bar{V}_S + jX\bar{I}$

(a) $\bar{I} = 1.0\angle{-30^\circ} \Rightarrow \bar{V}_{conv} = 1 + j\,0.05\,(1\angle{-30^\circ})$

$= 1 + 0.05\angle{90^\circ} \times 1\angle{-30^\circ}$

$= 1 + 0.05\angle{60^\circ}$

$= 1 + 0.05\,(0.5 + j\,0.866)$

$= (1 + 0.025) + j\,0.0433$

$= 1.026\angle{2.42^\circ}$ pu

(b) $\bar{I} = 1\angle{30^\circ}$

Following (a), $\bar{V}_{conv} = 1 + 0.05\angle{120^\circ}$

$= 1 + 0.05\,(-0.5 + j\,0.866)$

$= (1 - 0.025) + j\,0.0433$

$= 0.976\angle{2.54^\circ}$

(c) $\bar{I} = -1\angle{-30^\circ} \Rightarrow \bar{I} = 1\angle{180} \times 1\angle{-30^\circ}$

$= 1\angle{150^\circ}$

Following (a), $\bar{V}_{conv} = 1 + 0.05\angle{240^\circ}$

$= 1 + 0.05\angle{-120}$

$= 1 - 0.025 - j\,0.043$

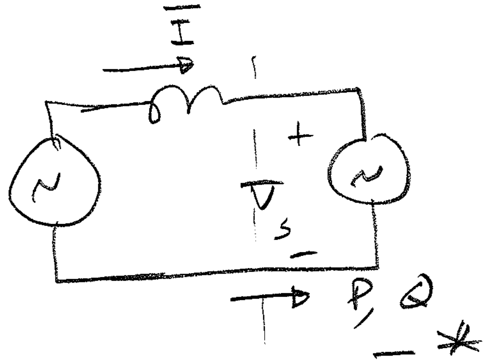
$= 0.976\angle{-2.52^\circ}$

(d) $\bar{I} = -1.0\angle{30^\circ} \Rightarrow \bar{I} = 1\angle{180+30} = 1\angle{-150^\circ}$

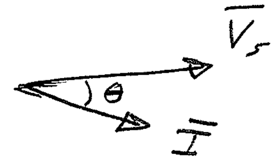
$\therefore \bar{V}_{conv} = 1 + 0.05\angle{-60^\circ} = 1 + 0.05\,(0.5 - j\,0.866)$

$= 1.025 - j\,0.0433 = 1.026\angle{-2.42^\circ}$

(b)



$\bar{V}_s = 1\angle 0^\circ$



$$P + jQ = \bar{V}_s \bar{I}^*$$
$$= \bar{I}^*$$

$\bar{I} = I\angle -\theta$

(1) $\bar{I} = 1\angle -30^\circ$ | $\bar{I}^* = 1\angle 30^\circ$ $= 0.866 + j0.5$

$\therefore$ $P = 0.866$ pu

$Q = 0.5$ pu

(2) $\bar{I} = 1\angle 30^\circ \Rightarrow \bar{I}^* = 1\angle -30^\circ$

$= 0.866 - j0.5$

$P = 0.866$, $Q = -0.5$ pu

(3) $\bar{I} = -1\angle 30^\circ = 1\angle 150^\circ$

$\therefore$ $\bar{I}^* = 1\angle -150^\circ$

$= -0.866 - j0.5$

$\therefore$ $P = -0.866$ pu, $Q = -0.5$ pu

(4) $\bar{I} = -1\angle 30^\circ = 1\angle -150^\circ$

$\bar{I}^* = 1\angle 150^\circ$

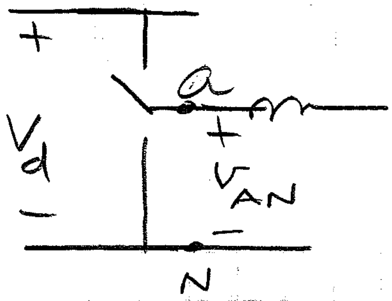
$= -0.866 + j0.5$

$\therefore$ $P = -0.866$ pu

$Q = +0.5$ pu

Chapter 4

4-1



$V_d = 150\,V$

$\hat{V}_{tri} = 5\,V$

$f_s = 20\,kHz$

$$d_a = \frac{\bar{v}_{aN}}{V_d} \Rightarrow v_{c,a} = d_a \hat{V}_{tri}$$

$$= (2d_A - 1)\hat{V}_{tri}$$

(a) $\bar{v}_{aN} = 125\,V$

$$\therefore d_a = \frac{\bar{v}_{aN}}{V_d} = 0.833$$

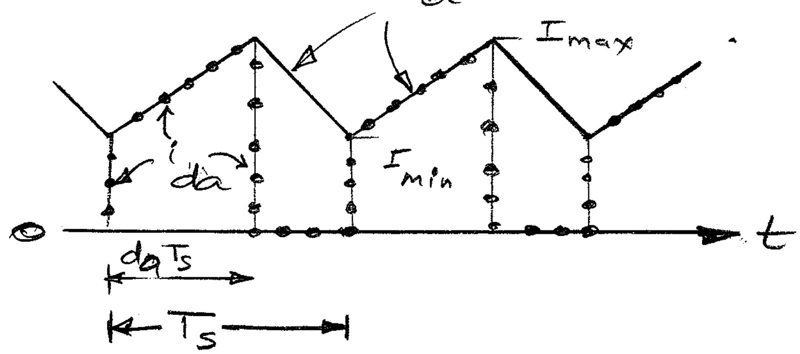
$$\therefore v_{c,a} = 0.833 \times 5 = 4.1667\,V$$

(b) $\bar{v}_{aN} = 50\,V$

$$\therefore d_a = \frac{\bar{v}_{aN}}{V_d} = 0.333$$

and $v_{c,a} = \cancel{1.665\,V} \; -1) \; 5.0 = -1.67\,V$

4-2



In general, assuming a ripple in i_a

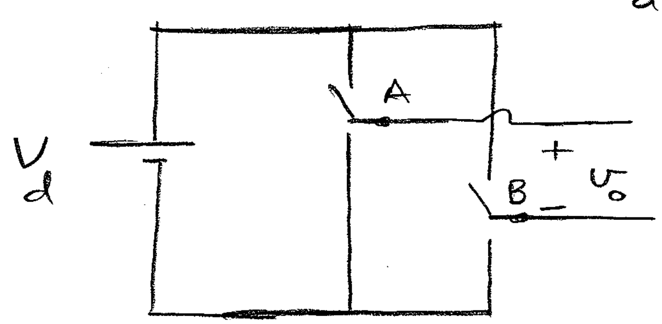
$$\bar{i}_a = \frac{I_{min} + I_{max}}{2}$$

$$\bar{i}_{da} = \frac{I_{min} + I_{max}}{2} d_a$$

$$\therefore \bar{i}_{da} = d_a \bar{i}_a$$

$\therefore$ i_{da} can be plotted for two values of $\bar{v}_{aN}$ with $d_a = 0.833$ and 0.333.

4-3



$V_d = 150\,V$

$f_s = 25\,kHz$

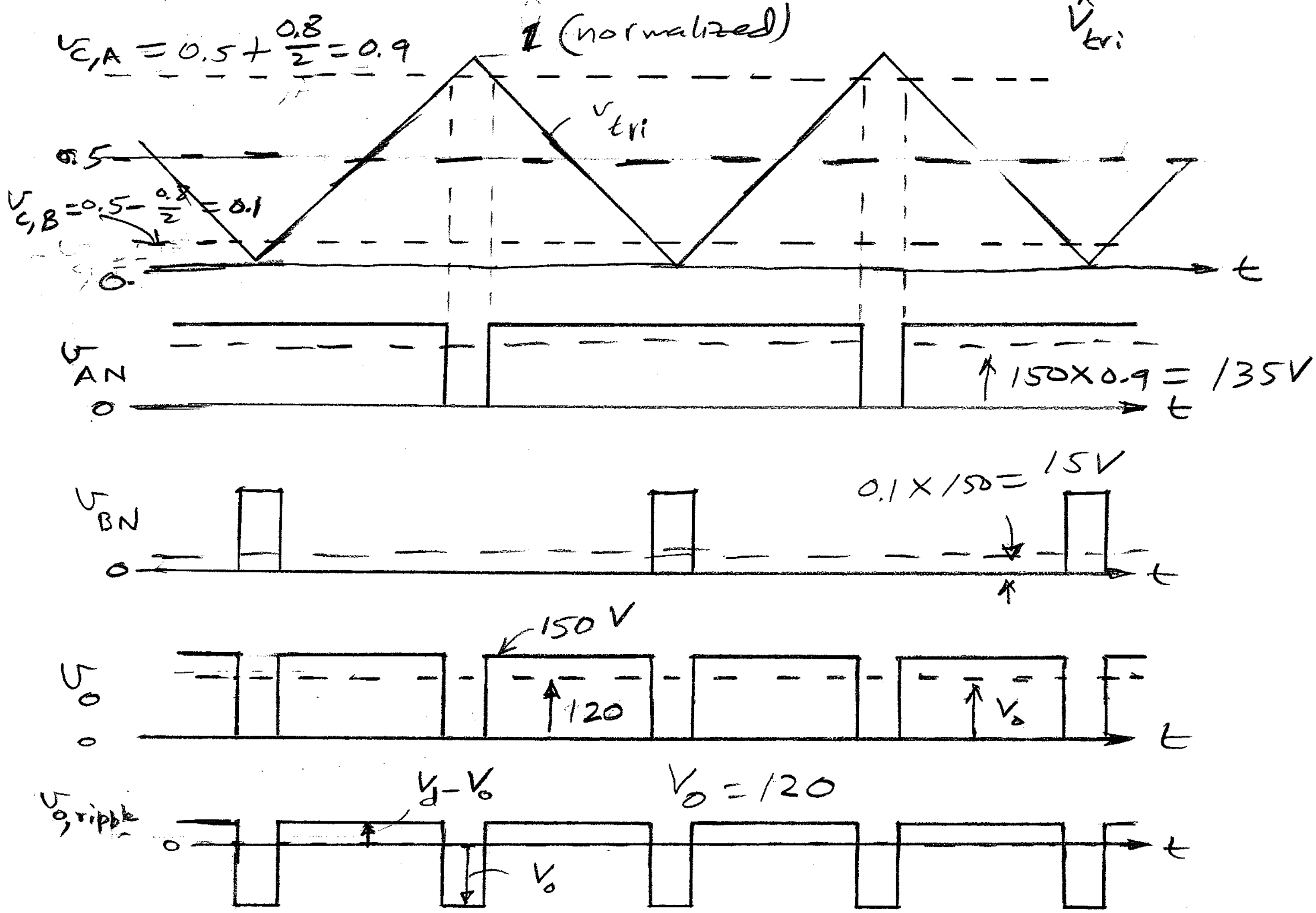
$\hat{V}_{tri} = 3V$

$$\bar{v}_o = k_{PWM} v_{control} \quad (4\text{-}19)$$

$$\text{where } k_{PWM} = \frac{V_d}{\hat{V}_{tri}} = \frac{150V}{3V} = 50 \quad (4\text{-}20)$$

4-4

$A \equiv a \qquad \dfrac{v_{control}}{\hat{V}_{tri}} = 0.8$

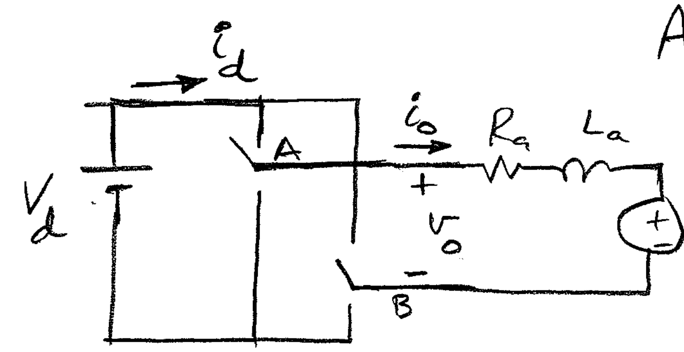


$$V_o = \frac{v_c}{\hat{V}_{tri}} V_d = 0.8 \times 150 = 120\ V$$

$$\therefore \quad V_d - V_o = 150 - 120 = 30\ V$$

4-5

$A \equiv a$



$f_s = 20$ kHz

$T_s = 50\ \mu s$

In steady state

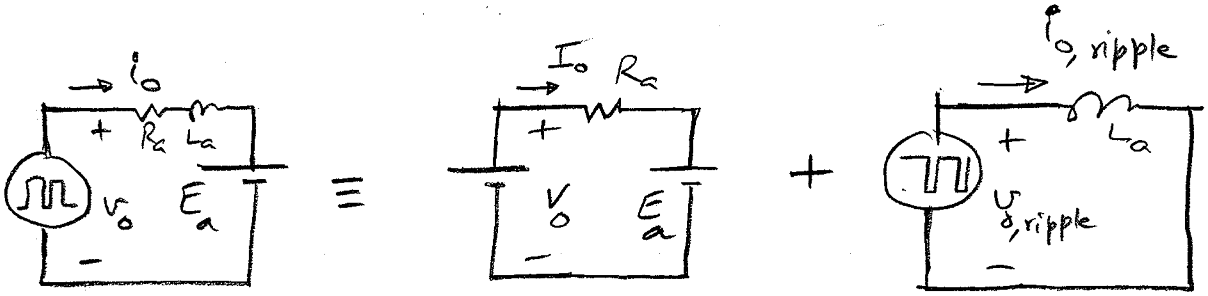
$$\overline{v}_o = V_o = E_a + R_a I_o = 100 + 0.25 \times 8$$
$$= 102 \text{ V}$$

$$\frac{v_c}{\hat{V}_{tri}} = \frac{\overline{v}_o}{V_d} = \frac{102}{150} = 0.68 = d$$

$$d_A = \frac{1}{2} + \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = 0.5 + 0.5 \times 0.68 = 0.84$$

and

$$d_B = \frac{1}{2} - \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = 0.5 - 0.5 \times 0.68 = 0.16$$

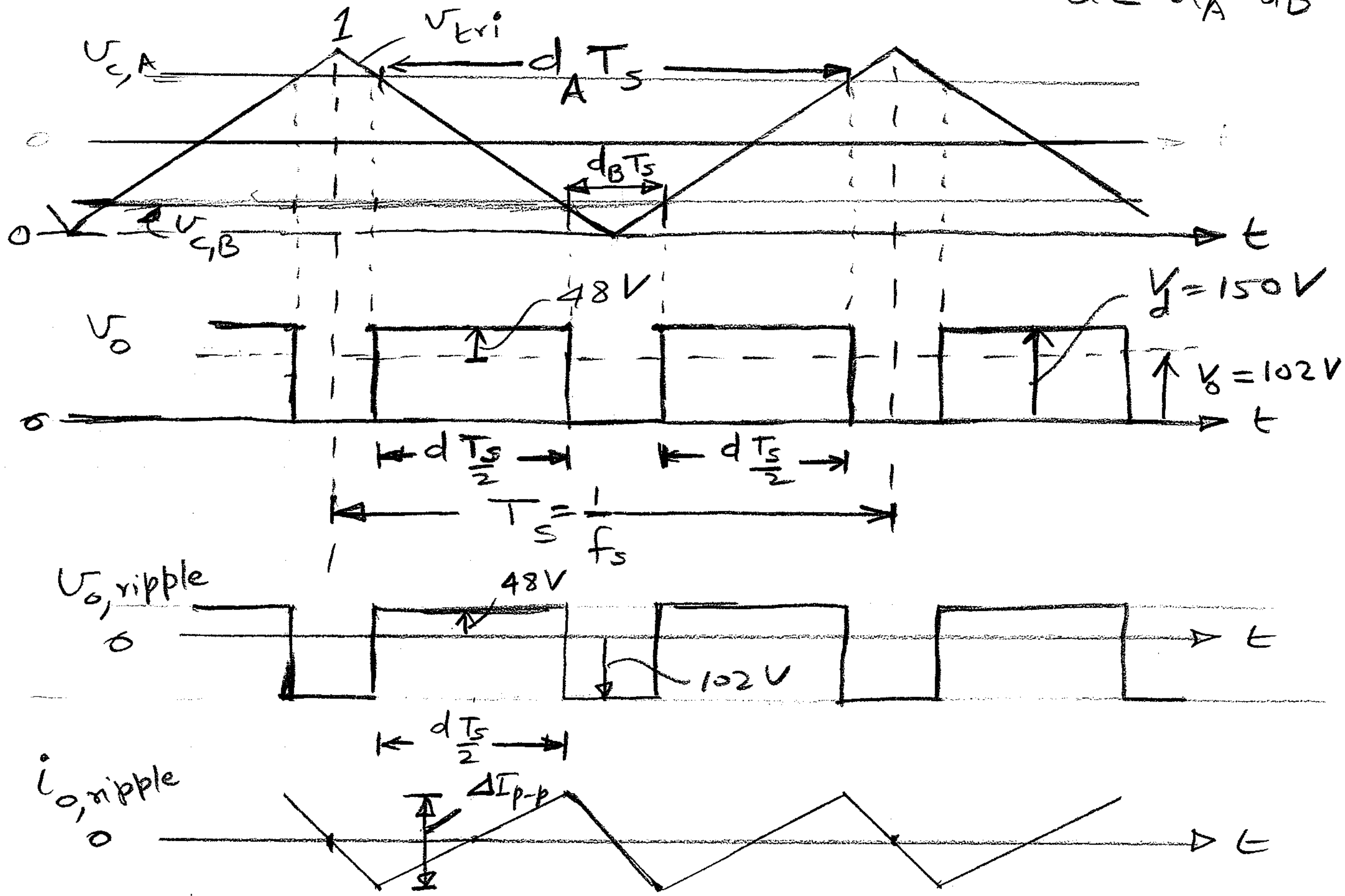


(dc)

ripple freq.
(R_a can be neglected)

and
$$i_o(t) = I_o + i_{o,\,ripple}$$
$$v_o(t) = V_o + v_{o,\,ripple}$$

Let $d = d_A - d_B$



$d = d_A - d_B$
$= 0.84 - 0.16$
$= 0.68$

Across the inductor, during the interval $d\frac{T_s}{2}$,

$$L \frac{\Delta I_{p-p}}{d\frac{T_s}{2}} = 150 - 102 = 48\,V$$

$$\therefore \quad \Delta I_{p-p} = \frac{48 \times 0.68 \times \frac{50 \times 10^{-6}}{2}}{4 \times 10^{-3}} = 0.204\,A$$

- $i_o(t)$ waveform is superposition of $i_{o,ripple}$ waveform on $I_o = 8\,A$.
- $i_d(t)$ waveform is similar to $i_o(t)$ waveform, except it goes to zero when $v_o(t)$ goes to zero.

4-6

Regenerative Braking mode

$$V_o = E_a + R_a I_a$$

$$= 100 + 0.25 \times (-8.0) = 98\ V$$

$$\frac{v_c}{\hat{V}_{tri}} = \frac{\overline{V_o}}{V_d} = \frac{98}{150} = 0.653$$

$$d_A = \frac{1}{2} + \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = \frac{1}{2}(1 + 0.653) = 0.827$$

and

$$d_B = \frac{1}{2} - \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = \frac{1}{2}(1 - 0.653) = 0.174$$

To calculate the peak-peak ripple in i_o:

$$L\frac{\Delta I_{p-p}}{d\frac{T_s}{2}} = 150 - 98 = 52\ V \qquad \text{where } d = d_A - d_B = 0.653$$

$$\therefore\ \Delta I_{p-p} = \frac{52 \times 0.653 \times \frac{50\times10^{-6}}{2}}{4\times10^{-3}} = 0.212\ A$$

The waveforms of v_o, v_o, ripple and i_o, ripple are nearly identical to those in Problem 4-5, except drawn for $d = 0.653$ (rather than $d = 0.68$).

The output current i_o has a waveform

that is now obtained by superimposing $i_{o,\,ripple}$ on -8.0 A. The same comment, as in Problem 4-5 applies to the $i_d(t)$ waveform.

Average Power Flow Into the Converter:

Power supplied by E_a is

$$P_{E_a} = E_a \times (I_o)_{avg} = 100 \times 8 = 800 \text{ W}.$$

Some of this power is dissipated into R_a, which is

$$P_{diss} = R_a \, I_{o,rms}^2$$

Including the ripple,

$$i_o = I_{o,avg} + \underbrace{\Sigma \, i_h}_{i_{o,\,ripple}}$$

For estimation purposes, we can ignore the effect of $i_{o,ripple}$ here (since its peak-peak value is much smaller than $I_{o,avg}$). Therefore,

$$I_{o,rms}^2 \simeq I_{o,avg}^2 = 8^2 = 64 \text{ A}^2$$

and $P_{diss} = 0.25 \times 64 = 16$ W

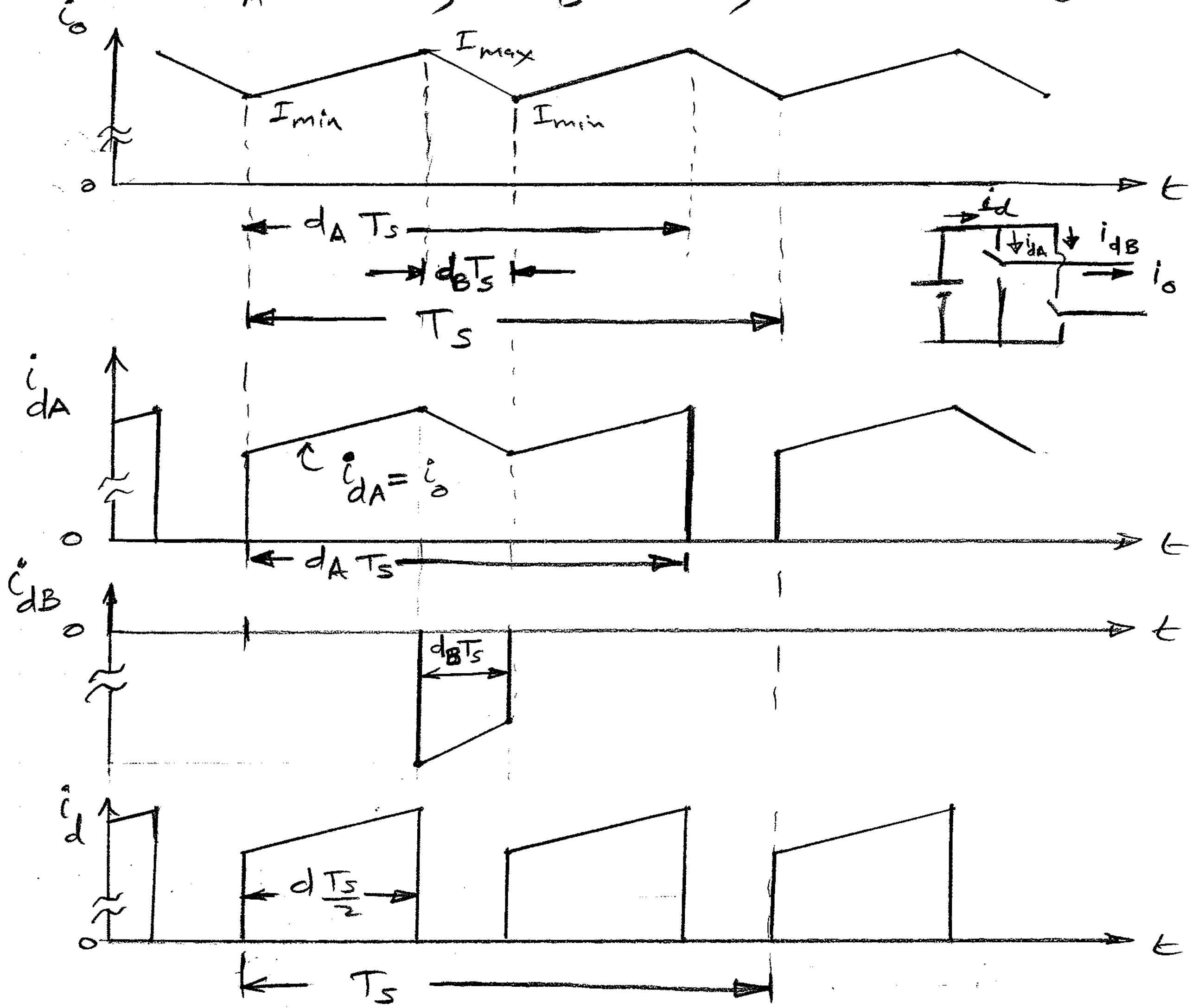
$\therefore$ The average power into the converter is

$$P_{conv} = P_{E_a} - P_{diss} = 800 - 16 = 784 \text{ W}$$

4-7

In Problem 4-5

$d_A = 0.84, \quad d_B = 0.16, \quad d = 0.68$



$I_{max} = I_o + \frac{\Delta I_{p-p}}{2}, \quad I_{min} = I_o - \frac{\Delta I_{p-p}}{2}$

It is easy to calculate $\bar{i}_{dB}$ and $\bar{i}_d$:

$$\bar{i}_{dB} = -\frac{I_{min} + I_{max}}{2} \cdot d_B = -I_o d_B = -8 \times 0.16 = -1.28\,A$$

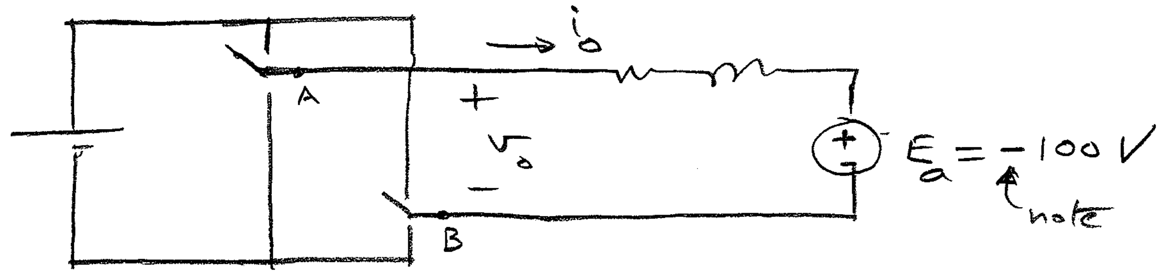
$$\bar{i}_d = \frac{I_{min} + I_{max}}{2} d = I_o \cdot d = 8.0 \times 0.68 = 5.44\,A$$

By Kirchoff's Current Law: $i_d = i_{dA} + i_{dB} \quad \therefore \bar{i}_d = \bar{i}_{dA} + \bar{i}_{dB}$

$$\therefore \bar{i}_{dA} = \bar{i}_d - \bar{i}_{dB} = 5.44 - (-1.28) = 6.72\,A$$

[Note: $\bar{i}_{dA} = d_A I_o = 0.84 \times 8 = 6.72\,A$]

4-8



Note that v_o and i_o are defined with the same polarity and direction as in Problem 4-5.

$$\bar{v}_o = V_o = -102\ V$$

$$\therefore \quad \frac{v_c}{\hat{V}_{tri}} = \frac{\bar{v}_o}{V_d} = \frac{-102}{150} = -0.68 = d$$

note that d can become negative in a range: $-1 \leq d \leq 1$

$$d_A = \frac{1}{2} + \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = 0.5(1 - 0.68) = 0.16$$

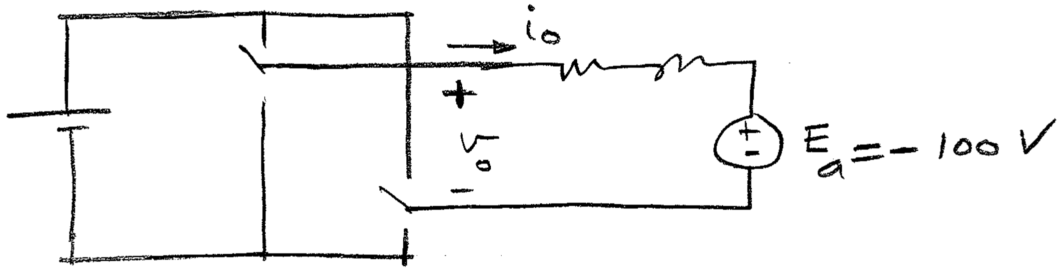
and $$d_B = \frac{1}{2} - \frac{1}{2}\frac{v_c}{\hat{V}_{tri}} = 0.5(1 + 0.68) = 0.84$$

Therefore, compared to Problem 4-5, the roles of poles A and B are interchanged here. However, $\Delta I_{p\text{-}p}$ ($= 0.204\,A$) would be the same. $I_{o,avg} = -8.0$. The waveform for v_o would now pulsate between 0V and −150V where,

$$V_o = E_a + R_a I_o$$

$$= -100 + 0.25 \times (-8.0) = -102\ V$$

4-9



The machine goes into regenerative braking mode while rotating in the reverse direction. Therefore, with the direction for i_o shown above,

$$I_o = 8.0\ A \quad (\text{it is positive})$$

and,

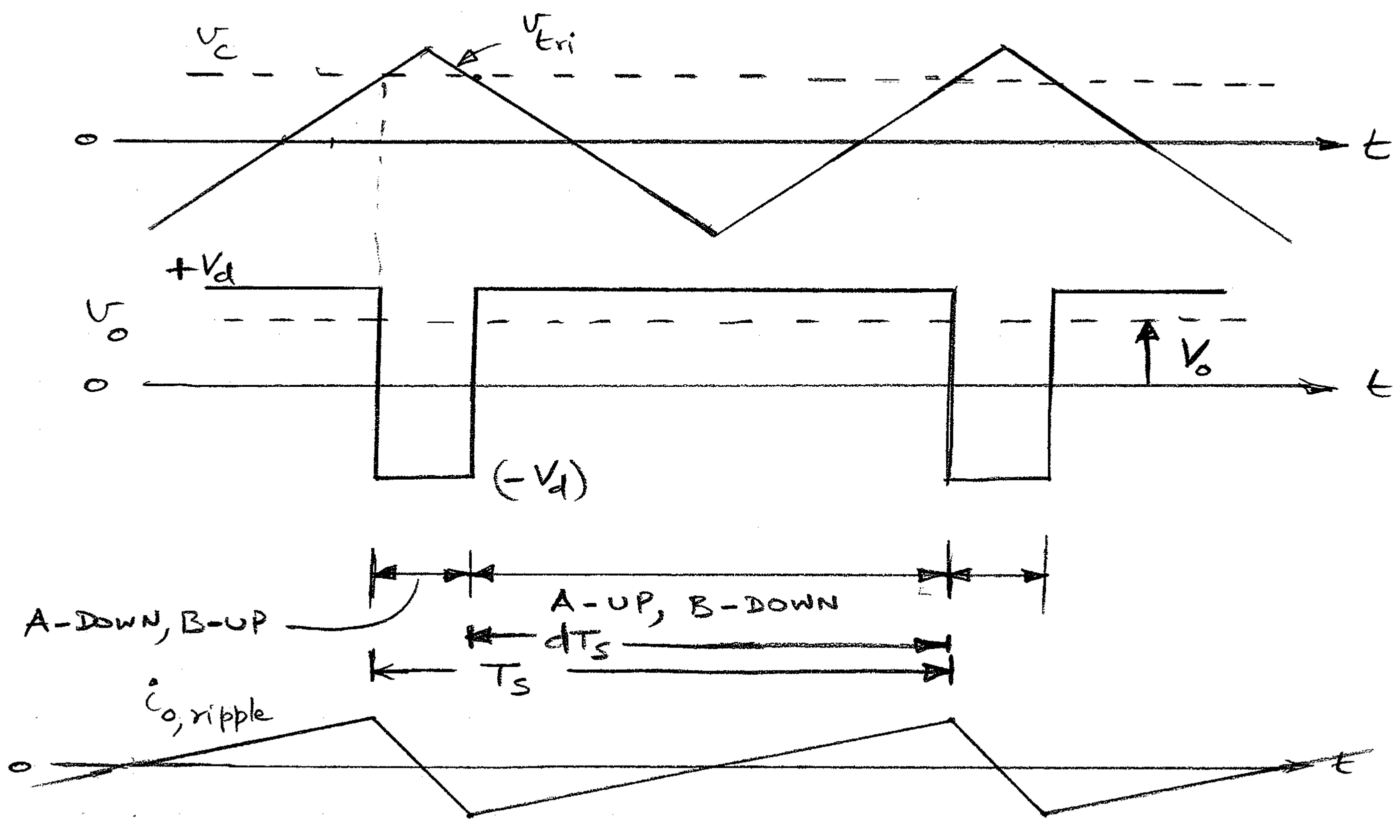
$$V_o = E_a + R_a I_o = -100 + 0.25 \times 8.0$$

$$= -98\ V$$

$$\Delta I_{p\text{-}p} = 0.281\ A \quad (\text{as in Problem } 4\text{-}6)$$

The various waveforms can be obtained using the previous problems.

4-10



During dT_s, $v_o = +V_d$

During $(1-d)T_s$, $v_o = -V_d$

$$\therefore \quad \overline{V_o} = \left[V_d \, d T_s + (-V_d)(1-d)T_s \right] / T_s$$

$$= (2d-1) V_d$$

From Problem 4-5, $\overline{V_o} = V_o = 102 \text{ V}$

$$\therefore \quad (2d-1) = \frac{\overline{V_o}}{V_d} = \frac{102}{150} = 0.68$$

$\therefore \quad d = 0.84$ which is equal to d_A in Problem 4-5.

To Calculate ΔI_{p-p}, notice that a voltage of $(V_d - V_o)$ appears across the inductor L_a for a period of $d\,T_s$

$$\therefore \quad L_a \frac{\Delta I_{p-p}}{d\,T_s} = V_d - V_o$$

or,

$$\Delta I_{p-p} = \frac{(V_d - V_o)\, d\, T_s}{L_a}$$

$$= \frac{(150-102) \times 0.84 \times 50 \times 10^{-6}}{4 \times 10^{-3}}$$

$$= 0.504 \text{ A}$$

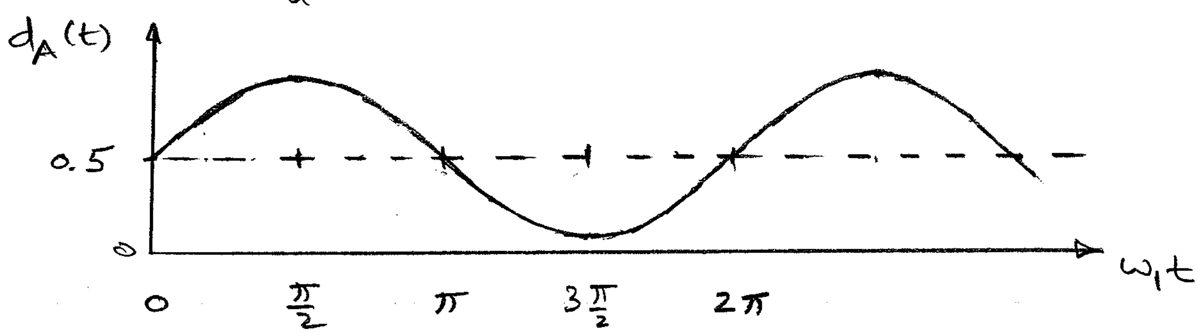
Notice that ΔI_{p-p} in a bi-polar switching dc-dc converter is more than twice that of Problem 4-5.

4-11 $\qquad A \equiv a$

$$\bar{v}_{AN} = \frac{V_d}{2} + 0.85 \frac{V_d}{2} \sin \omega_1 t$$

and,

$$d_A = \frac{\bar{v}_{AN}}{V_d} = \frac{1}{2} + \frac{0.85}{2} \sin \omega_1 t = 0.5 + 0.425 \sin \omega_1 t$$



4-12

$A \equiv a$

$V_d = 300\text{ V}, \quad \hat{V}_{tri} = 1\text{V}, \quad \bar{v}_{an}(t) = 90 \sin \omega_1 t \qquad f_1 = 45\text{ Hz}$

$\bar{v}_{aN} = \frac{V_d}{2} + \bar{v}_{an} = 150 + 90 \sin \omega_1 t,$

Similarly, $\bar{v}_{bN}$ and $\bar{v}_{cN}$.

$$d_A = \frac{\bar{v}_{aN}}{V_d}$$

$\therefore \quad d_A = 0.5 + 0.3 \sin \omega_1 t$

$d_B = 0.5 + 0.3 \sin(\omega_1 t - 120^\circ)$

$d_C = 0.5 + 0.3 \sin(\omega_1 t - 240^\circ)$

$\bar{v}_{AN} = d_A V_d = \frac{V_d}{2} + V_d \times 0.3 \sin \omega_1 t$

$\therefore \quad \bar{v}_{AN} = 150 + 90 \sin \omega_1 t$; etc. (checks)

and $\quad \bar{v}_{An} = 90 \sin \omega_1 t$; etc. (checks)

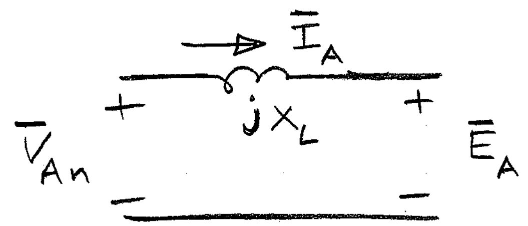
All these can be plotted similar to that in Problem 4-11.

Note that $\bar{v}_{An}$ is amplified v_{sA} by a gain of $112.5/0.75 = 150$ which equal to $\frac{1}{2}\frac{V_d}{\hat{V}_{tri}}$.

4-13

$$\bar{v}_{An}(t) = \frac{V_d}{2} 0.75 \sin \omega_1 t = 112.5 \sin \omega_1 t \ V$$

$$e_A(t) = 106.14 \sin(\omega_1 t - 6.6°) \ V$$



for purposes of drawing phasors, we will advance both voltages by 90° so that

$$\bar{v}_{An}(t) = 112.5 \cos \omega_1 t$$

and,

$$e_A(t) = 106.14 \cos(\omega_1 t - 6.6°)$$

$$\therefore \quad \bar{V}_{An} = 112.5 \angle 0° \ V, \quad \bar{E}_A = 106.14 \angle{-6.6°} \ V$$

$$\bar{V}_{An} = \bar{E}_A + j X_L \bar{I}_A \qquad X_L = 2\pi f_1 L$$

$$= 2\pi \times 45 \times 5 \times 10^{-3}$$

$$= 1.414 \ \Omega$$

$$\bar{I}_A = \frac{\bar{V}_{An} - \bar{E}_A}{j X_L}$$

$$= \frac{112.5\angle 0° - 106.14 \angle{-6.6°}}{j\, 1.414} = 9.97 \angle{-30.1°}$$

$$\therefore \quad \bar{i}_A(t) = 9.97 \cos(\omega_1 t - 30.1°) \quad \text{but in fact,}$$

$\bar{i}_A(t) = 9.97 \sin(\omega_1 t - 30.1°)$, taking away the advance by 90°.

$$d_A = \frac{1}{2} + \frac{1}{2} \times 0.75 \sin \omega_1 t = 0.5 + 0.375 \sin \omega_1 t$$

$$\bar{i}_{dA}(t) = d_A(t)\ \bar{i}_A(t)$$

$$= \left(0.5 + 0.375 \sin \omega_1 t\right) 9.97 \sin(\omega_1 t - 30.1°)$$

$$= 4.985 \sin(\omega_1 t - 30.1°) + 3.74 \sin \omega_1 t \cdot \sin(\omega_1 t - 30.1°)$$

$$\sin A \cdot \sin B = \frac{1}{2}\left\{\cos(A-B) - \cos(A+B)\right\}$$

$$\therefore\ \bar{I}_{dA} = 4.985 \sin(\omega_1 t - 30.1°) + 1.87 \cos(30.1°) - 1.87 \cos(2\omega_1 t - 30.1°)$$

$$= \underbrace{1.618}_{dc} + \underbrace{4.985 \sin(\omega_1 t - 30.1°)}_{\text{fund. freq.}} - \underbrace{1.87 \cos(2\omega_1 t - 30.1°)}_{2^{nd}\ \text{harmonic}}$$

4-14

$$\bar{i}_d = \bar{I}_{dA} + \bar{I}_{dB} + \bar{I}_{dC}$$

The fundamental frequency components in $\bar{I}_{dA}$, $\bar{I}_{dB}$ and $\bar{I}_{dC}$ combine to zero. The same happens to the 2nd harmonic components. Therefore,

$$\bar{I}_d = 3 \times 1.618 = 4.854\ \text{A}$$

Chapter 5

5-1

$$OD = 5.5\text{ cm}, \quad l_{OD} = \pi(OD) = 0.173\text{ m}$$

(a) $$\therefore H_{OD} = \frac{Ni}{l_{OD}} = \frac{25 \times 3}{0.173} = 434.1\text{ A/m}$$

$$ID = 5.0\text{ cm}, \quad l_{ID} = \pi(ID) = 0.157\text{ m}$$

(b) $$\therefore H_{ID} = \frac{Ni}{l_{ID}} = \frac{25 \times 3}{0.157} = 477.5\text{ A/m}$$

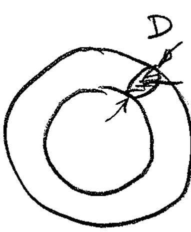
(c) From Example 5-1, $H_m = 454.5$ A/m

$$\%\ Error_{OD} = \frac{434.1 - 454.5}{454.5} = -4.5\%$$

$$\%\ Error_{ID} = \frac{477.5 - 454.5}{454.5} = 5.1\%$$

5-2

$$\mathcal{R}_m = \frac{l_m}{\mu_m A_m}$$



$$A_m = \frac{\pi D^2}{4} = \frac{\pi}{4}\left[\frac{OD - ID}{2}\right]^2 = 4.91 \times 10^{-6}\text{ m}^2$$

$$\therefore \mathcal{R}_m = \frac{0.165}{\mu_o \times 2000 \times 4.91 \times 10^{-6}} = 13.37 \times 10^6\ \frac{A}{Wb}$$

5-3

$L = 25\,\mu H$

$I_{max} = 3A, \quad B_{max} = 1.3T$

$H = ? \quad \mu_r = ?$

$$\mathcal{R}_m = \frac{0.165}{\mu_r \times \mu_o \times \underbrace{4.91 \times 10^{-6}}_{\text{from Problem 5-2}}}$$

$$L = \frac{N^2}{\mathcal{R}_m} = \frac{N^2 \mu_r \times \mu_o \times 4.91 \times 10^{-6}}{0.165} = 25\,\mu H \qquad (1)$$

At 3A, $\quad H_{max} = \dfrac{3N}{0.165}$

and, $\quad \mu_o \mu_r \dfrac{3N}{0.165} = \underbrace{1.3}_{B_{max}}$

or $\quad (\mu_r N) = \dfrac{1.3 \times 0.165}{\mu_o \times 3} \qquad (2)$

Substituting Eq. 2 into Eq. 1

$$25 \times \cancel{10^{-6}} = N \left(\frac{1.3 \times 0.165}{3 \cancel{\mu_o}} \right) \frac{\cancel{\mu_o} \times 4.91 \times \cancel{10^{-6}}}{0.165}$$

$$N = \frac{25 \times 0.165 \times 3}{1.3 \times 0.165 \times 4.91} = 11.75 \simeq 12 \text{ turns}$$

Therefore, from Eq. 2

$$\mu_r = \frac{1.3 \times 0.165}{\mu_o \times 3 \times 12} \cong 4{,}740$$

5-4

$$L = 25\,\mu H$$

$$I_{max} = 3.0\,A, \quad B_{max} = 1.3\,T$$

$$\mu_{core} = \infty$$

$A_g = A_m = 4.91\times10^{-6}\ m^2$ form Problem 5-2

In the air gap

$$\mu_o H_g = B_g = 1.3\,T$$

$$\therefore \quad \mu_o\left(\frac{3N}{l_g}\right) = 1.3$$

or

$$\left(\frac{N}{l_g}\right) = \frac{1.3}{3\mu_o} \qquad (1)$$

$$\mathcal{R} = \mathcal{R}_g = \frac{l_g}{\mu_o A_g}$$

$$L = \frac{N^2}{\mathcal{R}} = \frac{N^2 \mu_o A_g}{l_g} \qquad (2)$$

Substituting from Eq. (1) into (2)

$$25\times10^{-6} = N\left(\frac{1.3}{3\cancel{\mu_o}}\right)\cancel{\mu_o} A_g$$

$$\therefore \quad N = \frac{25\times10^{-6}\times3}{1.3\times A_g} = \frac{25\times10^{-6}\times3}{1.3\times4.91\times10^{-6}} \simeq 12 \text{ turns}$$

From Eq.(1)

$$l_g = \frac{3N\mu_o}{1.3} = 0.035\ mm$$

5-5

In Example 5-4

$$H_m = \frac{Ni}{l_m} = \frac{100 \times i}{0.18}$$

$$B_m = \mu_m H_m = \mu_o \times 5000 \times \frac{100 \times i}{0.18} = 0.3\,T$$

↑ given

$$\therefore \; i_{max} = \frac{0.3 \times 0.18}{\mu_o \times 5000 \times 100} = 86 \text{ mA.}$$

5-6

$$\mu_{core} = \infty, \quad l_g = 0.05 \text{ mm}$$

$$A_g = A_m = 5 \times 10^{-3} \times 15 \times 10^{-3} = 75 \times 10^{-6} \text{ m}^2$$

$$N = 100$$

(a) $$R = R_g = \frac{l_g}{\mu_o A_g}$$

$$L = \frac{N^2}{R} = \frac{100^2 \mu_o A_g}{l_g} = \frac{100^2 \times 4\pi \times 10^{-7} \times 75 \times 10^{-6}}{0.05 \times 10^{-3}}$$

$$= 18.85 \text{ mH}$$

(b) $$H_g = \frac{Ni}{l_g} = \frac{100 \times i}{0.05 \times 10^{-3}}$$

$$B_{core} = B_g = \mu_o H_g = \frac{4\pi \times 10^{-7} \times 100\, i_{max}}{0.05 \times 10^{-3}} = 0.3\,T$$

$$\therefore \; i_{max} = 119 \text{ mA}$$

5-7

$$B_{core} = B_g = 0.3\ T$$

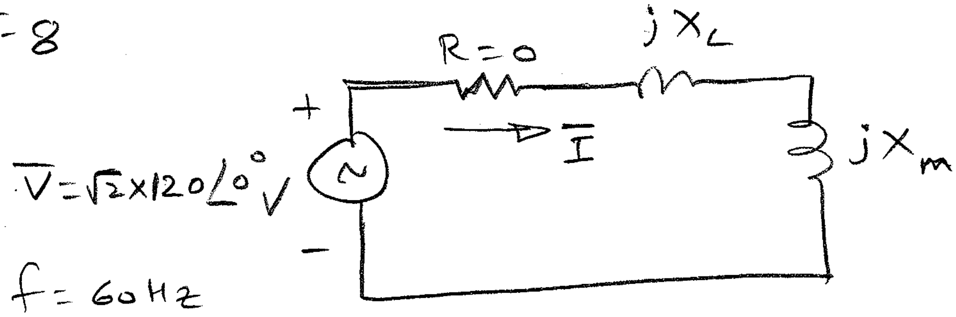
In the airgap, $w_g = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{0.3^2}{\mu_0}$

$$W = (vol)_{gap}\, w_g = \underbrace{75\times10^{-6}}_{area} \times \underbrace{0.05\times10^{-3}}_{l_g} \times \frac{1}{2}\frac{0.3^2}{\mu_0}$$

$$= 134\ \mu J$$

In the core, the energy stored is zero due to the assumption of $\mu_{core} = \infty$.

5-8



$$\bar{I} = \frac{\bar{V}}{j\omega(1.0+200)\times10^{-3}}$$

$$= \frac{\sqrt{2}\times120}{j\,2\pi\times60\times201\times10^{-3}} = 2.24\angle{-90^\circ}$$

$\therefore\ i(t) = 2.24\cos(\omega t - 90^\circ)\ A$

5-9

$$V_1 = N_1 \frac{d\phi}{dt}, \quad \text{Let } \phi = \hat{\phi} \sin \omega t$$

$$\therefore \frac{d\phi}{dt} = \omega \hat{\phi} \cos \omega t$$

$$\therefore V_1 = \underbrace{N_1 \omega \hat{\phi}}_{\hat{V}} \cos \omega t$$

at 60Hz

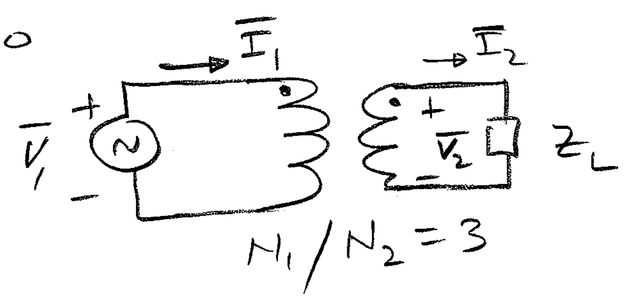
$$\phi_{max} = \frac{\hat{V}}{N_1 \omega} = \frac{\sqrt{2} \times 120}{N_1 \times 2\pi \times 60}$$

$\therefore$ At 50 Hz, for the same ϕ_{max}

$$\frac{\sqrt{2} \times V_{rms}}{N_1 \times 2\pi \times 50} = \underbrace{\frac{\sqrt{2} \times 120}{N_1 \times 2\pi \times 60}}_{\phi_{max}}$$

$$\therefore V_{rms}^{(50Hz)} = 120 \times \frac{50}{60} = 100V$$

5-10



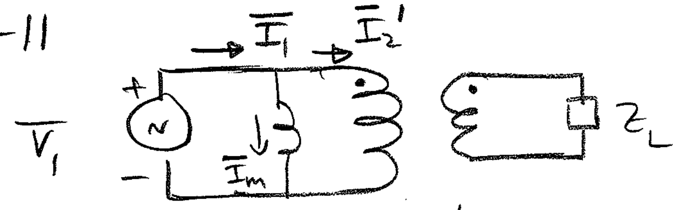
$$\bar{I}_2 = \frac{\bar{V}_2}{Z_L} = \frac{\bar{V}_1 (N_2/N_1)}{Z_L}$$

$$\bar{I}_1 = \frac{N_2}{N_1} \bar{I}_2$$

$$\therefore \bar{I}_1 = \frac{N_2}{N_1} \frac{\bar{V}_1}{Z_L} \frac{N_2}{N_1} = \left(\frac{N_2}{N_1}\right)^2 \frac{\bar{V}_1}{Z_L}$$

$$= \frac{\bar{V}_1}{Z_L \left(\frac{N_1}{N_2}\right)^2} = \frac{\sqrt{2} \times 120 \angle 0^\circ}{(5+j3) \times 9} = \sqrt{2} \times 2.29 \angle -31^\circ A$$

5-11



$N_1/N_2 = 3$

As shown in Problem 5-11, the impedance seen from side 1 is $\left(\frac{N_1}{N_2}\right)^2 Z_L$

Assuming $\bar{V}_1$ to be the reference phasor,

$$\bar{V}_1 = \sqrt{2} \times 120 \angle 0^\circ , \quad Z_L = 1.1 \angle 30^\circ$$

$$\bar{I}_2' = \frac{V_1}{\left(\frac{N_1}{N_2}\right)^2 Z_L} = \frac{\sqrt{2} \times 120}{(3)^2\, 1.1 \angle 30^\circ} = \sqrt{2} \times 12.12 \angle -30^\circ \text{ A}$$

$$\bar{I}_m = -j\sqrt{2} \times 1.0$$

$$\therefore \bar{I}_1 = \bar{I}_m + \bar{I}_2'$$

$$= \sqrt{2}\left[12.12 \angle -30^\circ - j1 \right] = \sqrt{2} \times 12.65 \angle -33.92^\circ \text{ A}$$

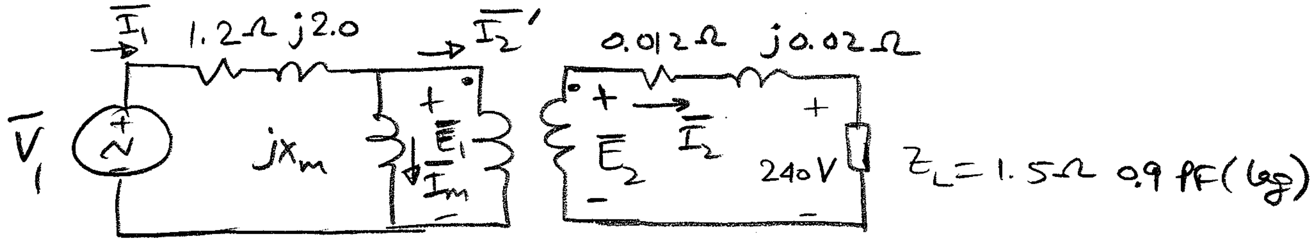
5-12

$\mu_{core} = 1/2$ of that in Problem 5-11

The core flux density remains the same. The magnetizing inductance will now be 1/2 as large as before, hence $\bar{I}_m = -j\sqrt{2} \times 2.0$ A. $\bar{I}_2'$ will be same as before. Therefore,

$$\bar{I}_1 = \sqrt{2}\left[12.12 \angle -30^\circ - j\,2\right] = \sqrt{2} \times 13.23 \angle -37.5^\circ \text{ A}$$

5-13



2400/240V

Let $\bar{V}_2$ be the reference phasor: $\bar{V}_2 = \sqrt{2} \times 240 \angle 0^\circ$ V

$$Z_L = 1.5 \angle 25.84^\circ \ \Omega$$

$$\bar{I}_2 = \frac{\bar{V}_2}{Z_L} = \sqrt{2}\ \frac{240 \angle 0^\circ}{1.5 \angle 25.84^\circ} = \sqrt{2} \times 160 \angle -25.84^\circ \ \text{A}$$

$$\bar{E}_2 = \bar{V}_2 + (0.012 + j0.02)\,\bar{I}_2$$
$$= \sqrt{2}\left[240 \angle 0^\circ + (0.012 + j0.02)\,160 \angle -25.84^\circ\right]$$

$$= \sqrt{2} \times 243.13 \angle 0.46^\circ \ \text{V}$$

$$\frac{N_1}{N_2} = \frac{2400}{240} = 10$$

$$\therefore\ \bar{I}_2' = \frac{N_2}{N_1}\,\bar{I}_2 = \sqrt{2} \times 16.0 \angle -25.84^\circ \ \text{A}$$

$$\bar{E}_1 = \frac{N_1}{N_2}\,\bar{E}_2 = \sqrt{2} \times 2431.3 \angle 0.46^\circ \ \text{V}$$

$$\therefore\ \bar{I}_m = \frac{E_1}{jX_m} = \sqrt{2} \times \frac{2431.3}{j1800} = \sqrt{2} \times 1.35 \angle -90^\circ \ \text{A}$$

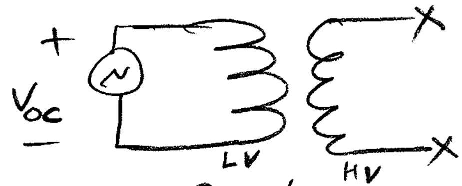
$$\therefore\ \bar{I}_1 = \bar{I}_2' + \bar{I}_m = \sqrt{2}\left[16 \angle -25.84^\circ + 1.35 \angle -90^\circ\right]$$
$$= \sqrt{2} \times 16.63 \angle -30^\circ$$

$$\therefore\ \bar{V}_1 = \bar{E}_1 + (1.2 + j2.0)\,\bar{I}_1 = \sqrt{2} \times \left[2431.3 \angle 0.46^\circ + (1.2 + j2.0) \times 16.63 \angle -30^\circ\right]$$

$$= \sqrt{2} \times 2465.4 \angle 0.9^\circ \ \underline{\underline{\text{V}}}$$

5-14

Open-circuit Test



240/2400 V

LV: low voltage, HV: High voltage

$$R_{he_{LV}} = \frac{V_{oc}^2}{P_{oc}} = \frac{240 \times 240}{400} = 144\,\Omega$$

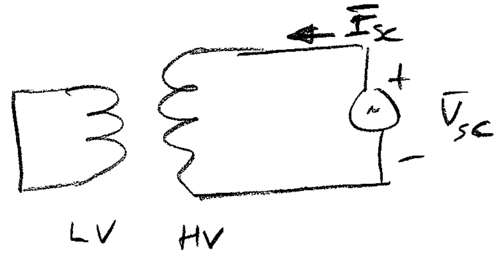
$$\frac{X_m R_{he}}{\sqrt{R_{he}^2 + X_m^2}} = \frac{V_{oc}}{I_{oc}} = \frac{240}{5.0} = 48\,\Omega$$

$$X_m^2 R_{he}^2 = 2304\,R_{he}^2 + 2304\,X_m^2$$

or $$X_m^2 (R_{he}^2 - 2304) = 2304\,R_{he}^2$$

$$\therefore X_{m_{LV}} = \sqrt{\frac{2304 \times 144^2}{144^2 - 2304}} = 50.91\,\Omega$$

Short-Circuit Test



$$R_{2_{HV}} = \frac{1}{2}\frac{P_{sc}}{I_{sc}^2}$$

$$= \frac{1}{2} \times \frac{700}{20^2}$$

$$= 0.875\,\Omega$$

$$R_{1_{LV}} = \left(\frac{240}{2400}\right)^2 \times R_{2_{HV}}$$

$$= 0.00875\,\Omega$$

$$(2R_2)^2 + (2X_{\ell_2})^2 = \left(\frac{V_{sc}}{I_{sc}}\right)^2$$

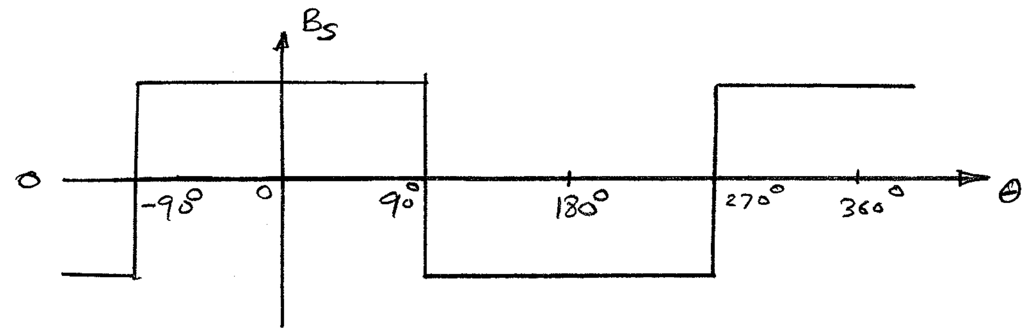
$$\therefore \quad (2X_{\ell_2})^2 = \left(\frac{90}{20}\right)^2 - (2R_2)^2 = 17.188$$

$$\therefore \quad X_{\ell_{2_{HV}}} = 2.07\ \Omega$$

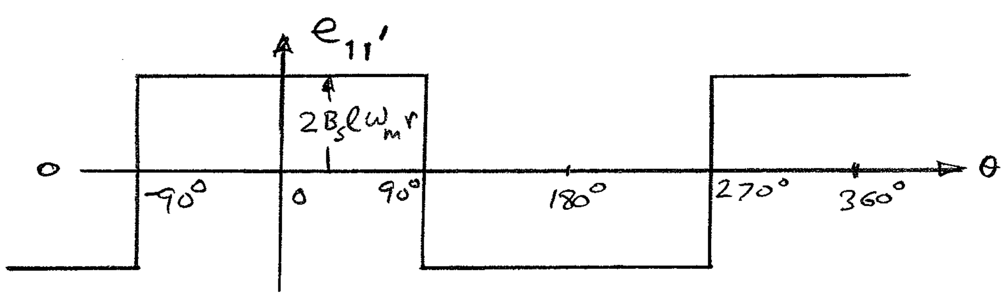
and, $X_{\ell_{1_{LV}}} = \left(\frac{240}{2400}\right)^2 X_{\ell_2} \simeq 0.02\ \Omega$

Chapter 6

6-1

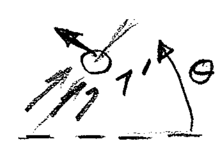


(a)



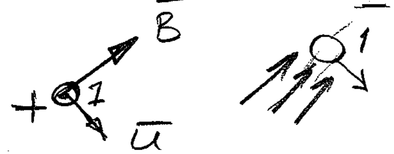
$e_{11'}$ waveform is shown above; it is independent of i_a.

Note the $e_{11'} = +$ in the position shown:



Conductor 1

Amplitude of $e_{11'} = 2B_s l \underbrace{(\omega_m r)}_{u}$



$f_q = q(\bar{u} \times \bar{B})$

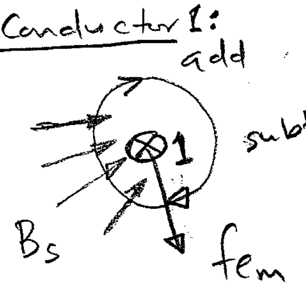
$\therefore$ 1 is positive at the near end.

$\therefore$ $e_{11'}$ is positive in the position shown, it will be so for $-90° \leq \theta \leq 90°$, as plotted above.

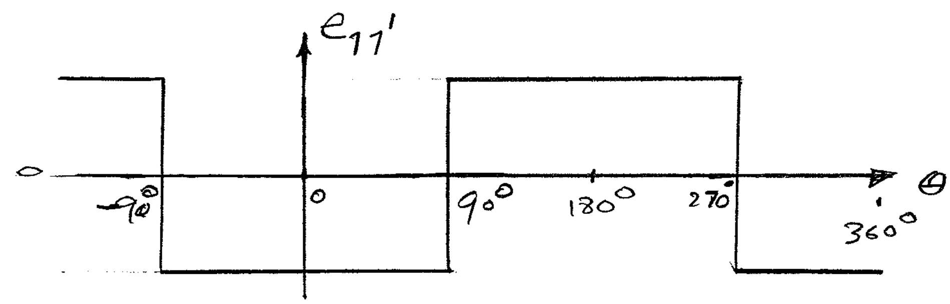
(b) $T_{em} = 2B_s I_o l r$, (CCW)

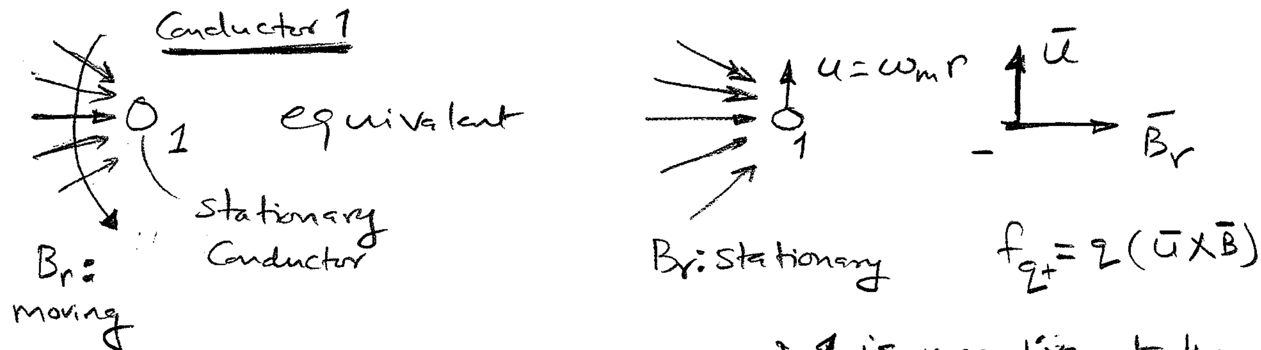
independent of ω_m.

Conductor 1:



6-2



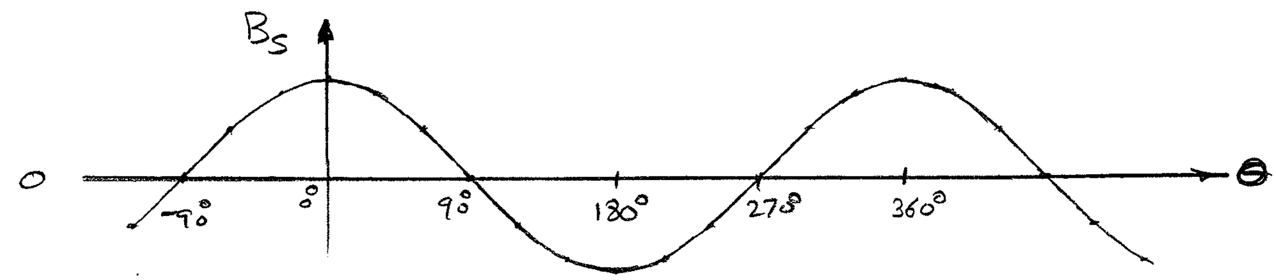


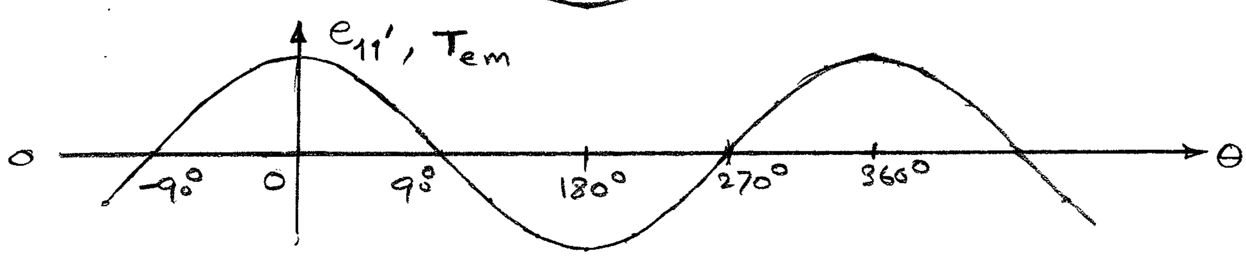
$\therefore$ 1 is negative at the near end.

Amplitude of $e_{11'} = 2B_r \, l \, (\omega_m r)$

$\therefore$ $e_{11'}$ is negative in the position shown; it will be so for $-90^\circ \leq \theta \leq 90^\circ$, as plotted above.

6-3





$$e_{11'}(\theta) = 2\hat{B}\cos\theta \, l \, \omega_m r \quad ; \quad T_{em}(\theta) = 2\hat{B}\cos\theta \, l \, I \, r$$

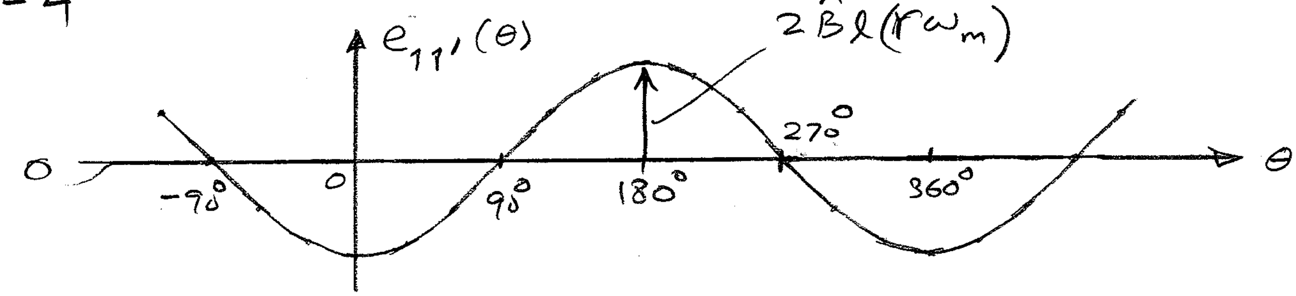
$$p_{el}(\theta) = e_{11'} I = 2I\hat{B}\cos\theta \, l \, \omega_m r$$

$$p_{mech}(\theta) = T_{em}\omega_m = 2I\hat{B}\cos\theta \, l \, \omega_m r$$

where $\omega_m = 60 \frac{rad}{s}$

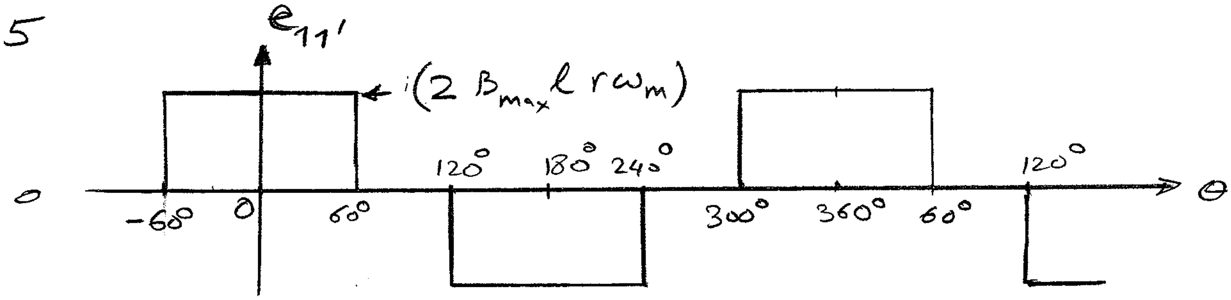
Note: Avgerage power $= 0$

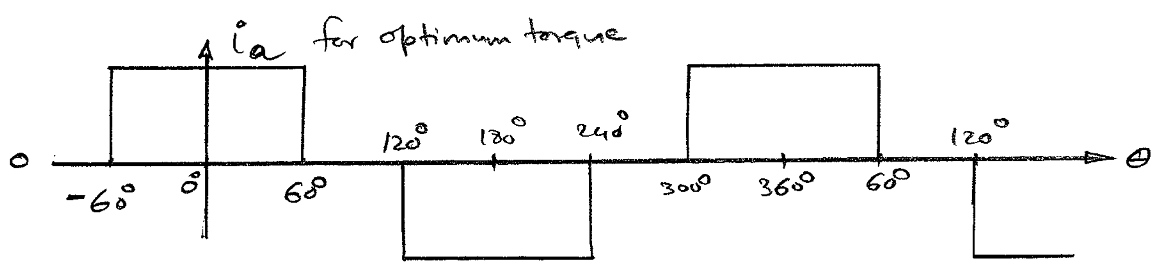
6-4

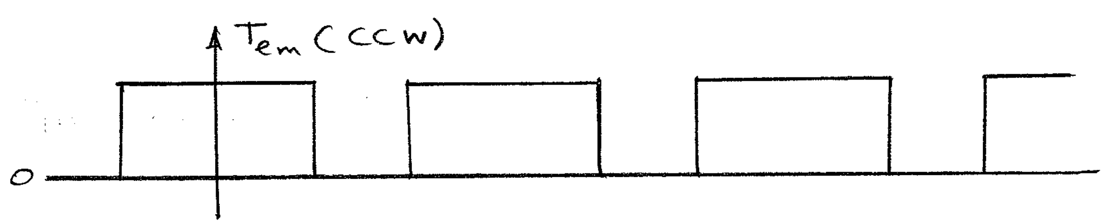


Note: similar to Problem 6-2, except for a sinusoidal flux-density distribution.

6-5







Notice that the torque waveform is pulsating (there is no way to avoid that in this structure); however it is always in the same direction.

6-6

In steady state, $E_{rod} = BLu = 1 \times 2 \times u = 2u$

$$I = \frac{50 - E_{rod}}{R} = \frac{50 - 2u}{0.1}$$

$$F_{em} = BIL = 1 \times \frac{50-2u}{0.1} \times 2$$
$$= 20 \times (50 - 2u) \quad N$$

In steady state, $F_{em} = F_u$

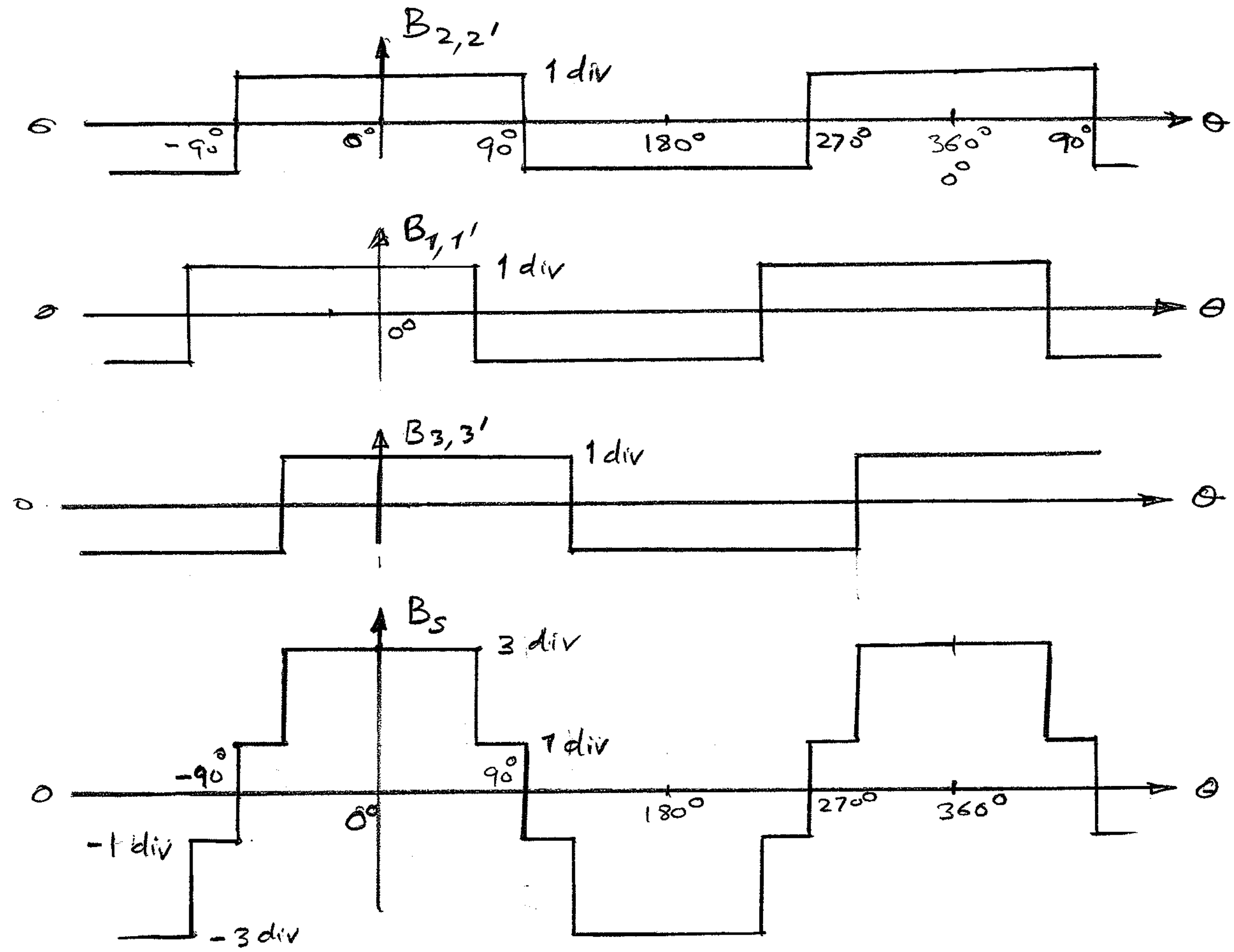
$\therefore$

$$20(50 - 2u) = 1500\,u^2$$

or, $$u^2 + \frac{40}{1500} u = \frac{1000}{1500} = 0.667$$

$\therefore \quad u \simeq 0.8$ m/s

6-7



6-8 In the position shown,

On the rotor,

$$T_r\ (ccw) = 2 B_s \ell N_r i_r$$

where, $B_s = \mu_o \dfrac{N_s i_s}{2\ell_g}$

$$\therefore T_r(ccw) = 2\left(\mu_o \frac{N_s i_s}{2\ell_g}\right) \ell N_r i_r$$

On the stator

$$T_s\ (CW) = 2 B_r \ell N_s i_s$$

where, $B_r = \mu_o \dfrac{N_r i_r}{2}$

$$\therefore T_s(CW) = 2\left(\mu_o \frac{N_r i_r}{2\ell_g}\right) \ell N_s i_s$$

$$\therefore |T_r| = |T_s| = \frac{\mu_o N_s N_r \ell i_s i_r}{\ell_g}$$

Problem 6-9

As discussed in this chapter, this would be the case if $\omega_m < \omega_{syn}$.

Chapter 7

7-1 $R_a = 0.35\Omega$, $L_a = 1.5\text{ mH}$, $k_E = k_T = 0.5$

$T_{rated} = 4\text{ Nm}$

(a) $V_a = 100V$

$$E_a\Big|_{no-load} = k_E \omega_m \Rightarrow \omega_m\Big|_{no-load} = \frac{E_a}{k_E} = \frac{100}{0.5} = 200\ \frac{rad}{s}$$

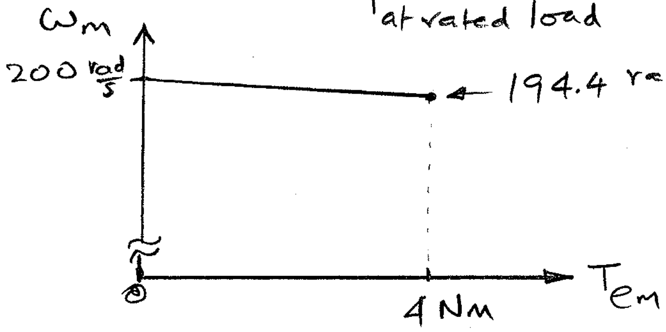
at $T_{rated} = 4\text{ Nm}$ $\Rightarrow$ $I_{a,rated} = \frac{T}{k_T} = \frac{4.0}{0.5} = 8.0\text{ A}$

$$V_a = k_E \omega_m + R_a I_{a,rated}$$

$$\therefore \omega_m\Big|_{at\ rated\ load} = \frac{V_a - R_a I_{a,rated}}{k_E}$$

$$= \frac{100 - 0.35 \times 8.0}{0.5} = 194.4\ \frac{rad}{s}$$

$$\therefore \Delta\omega_m\Big|_{at\ rated\ load} = 200 - 194.4 = 5.6\ rad$$



(b) $V_a = 60V$

$$\omega_m\Big|_{no-load} = \frac{E_a}{k_E} = \frac{60}{0.5} = 120\ rad/s$$

$$\Delta\omega_m\Big|_{at\ rated\ load} = 5.6\ rad \quad \therefore \omega_m\Big|_{rated\ load} = 120 - 5.6 = 114.4\ \frac{rad}{s}$$

(c) $V_a = 30V$

$$\omega_m\Big|_{no\ load} = \frac{E_a}{k_E} = \frac{30}{0.5} = 60\ rad/s$$

$$\omega_m\Big|_{rated-load} = 60 - 5.6 = 54.4\ \frac{rad}{s}$$

7-2

$$\omega_m = 1500 \text{ rpm} = \frac{1500}{60} \times 2\pi = 157.08 \ \frac{rad}{s}$$

$$T_{em} = 3 \text{ Nm} \Rightarrow I_a = \frac{T_{em}}{k_T} = \frac{3}{0.5} = 6.0 \text{ A}$$

$$\therefore \quad V_a = k_E \omega_m + R_a I_a = (0.5 \times 157.08) + (0.35 \times 6.0)$$

$$= 80.64 \text{ V}$$

7-3

$$\omega_m = 1500 \text{ rpm} = \frac{1500}{60} \times 2\pi = 157.08 \ \frac{rad}{s}$$

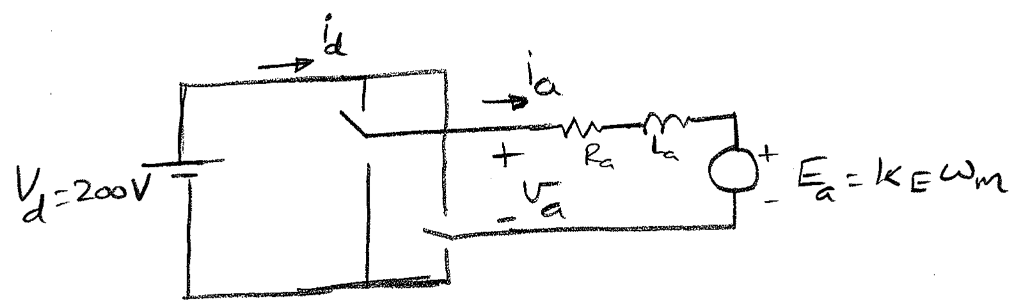
Regen. Braking $I_a = -10$ A

$$V_a = k_E \omega_m + R_a I_a$$

$$= (0.5 \times 157.08) + 0.35(-10.0)$$

$$= 75.04 \text{ V}$$

7-4



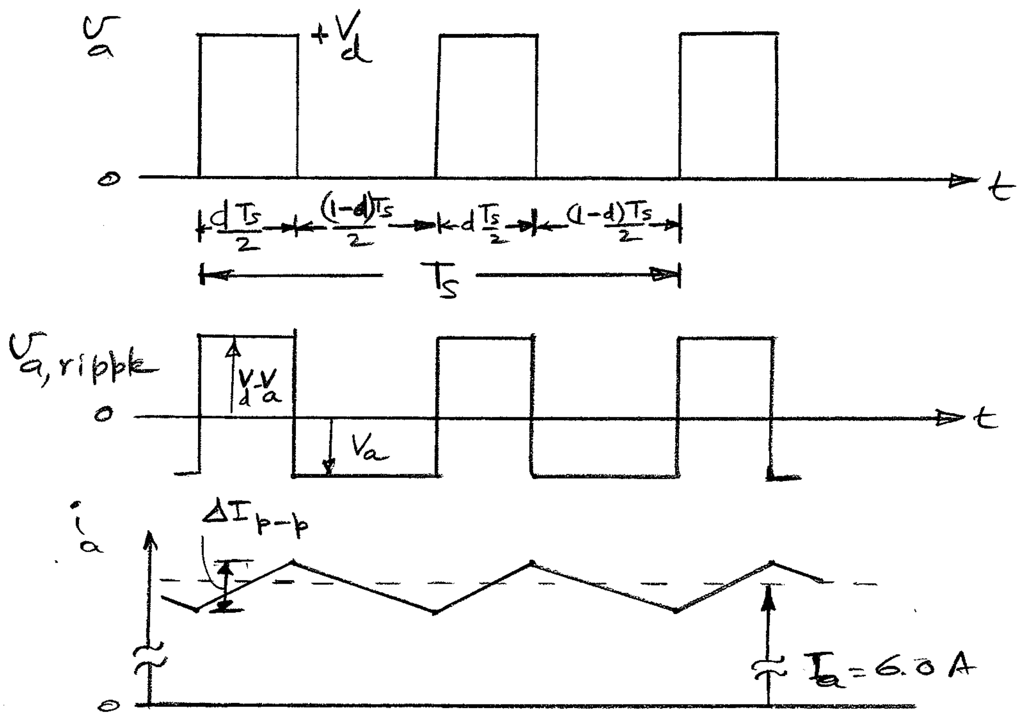
(a) $\omega_m = 1500 \text{ rpm} = \frac{1500}{60} \times 2\pi = 157.08 \ \frac{rad}{s}$

$$T_{em} = 3 \text{ Nm} \Rightarrow I_a = \frac{T_{em}}{k_T} = \frac{3.0}{0.5} = 6.0 \text{ A}$$

$$V_a = k_E \omega_m + R_a I_a = 80.64 \text{ V}$$

From Eq. [illegible] $\quad V_a = \frac{V_c}{\hat{V}_{tri}} V_d \quad \therefore \quad \frac{V_c}{\hat{V}_{tri}} = \frac{V_a}{V_d} = \frac{80.64}{200} = 0.403 = d$

$(d_a - d_b)$



$$T_s = \frac{1}{f_s} = 40\mu s$$

$$L_a \frac{\Delta I_{p-p}}{d\frac{T_s}{2}} = V_d - V_a \quad \therefore \Delta I_{p-p} = \frac{1}{L_a}(V_d - V_a)\, d\frac{T_s}{2}$$

$$= \frac{1}{1.5\times10^{-3}}(200 - 80.64)\,\frac{0.403}{2}\times 40\times10^{-6}$$

$$= 0.641\,A$$

(b) $\omega_m = 1500$ rpm $= 157.08$ rad/s

$I_a = -10A$

$$V_a = k_E\omega_m + R_a I_a = 0.5\times157.08 + [0.35\times(-10)]$$

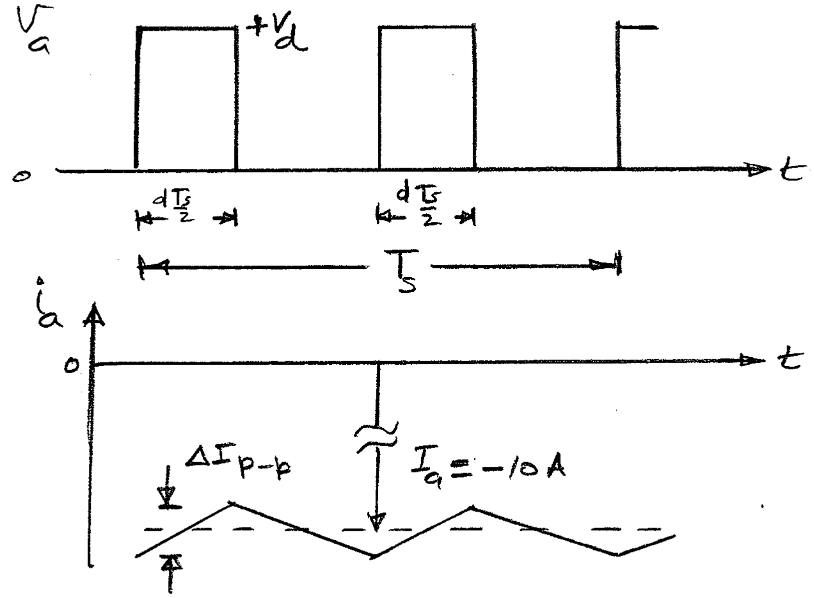
$$= 75.04\,V$$

$$d = \frac{V_a}{V_d} = \frac{75.04}{200} = 0.375$$

$$\Delta I_{p-p} = \frac{1}{L_a}(V_d - V_a)\, d\frac{T_s}{2}$$

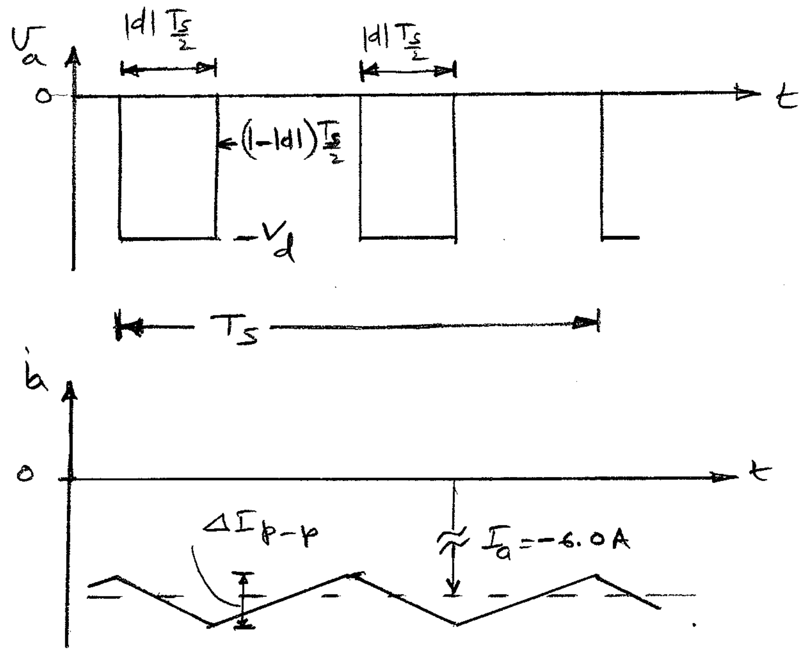
$$= \frac{1}{1.5\times10^{-3}}(200.0 - 75.04)\times\frac{0.375}{2}\times 40\times10^{-6}$$

$$= 0.625A$$



(c) $\omega_m = -157.08$ rad/s $\qquad I_a = -6.0\,A$

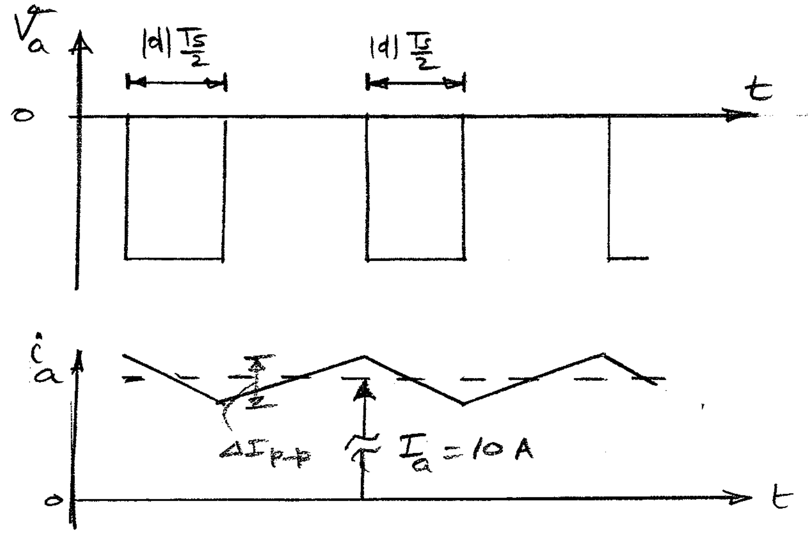
$V_a = -80.64\,V$, $\quad d = -0.403$, $\quad \Delta I_{pp} = 0.641\,A$ as in part a



(d) $\omega_m = -157.08 \text{ rad/s}$, $I_a = +10 \text{ A}$

$$V_a = k_E \omega_m + R_a I_a$$
$$= -75.04 \text{ V}$$

$d = \dfrac{V_a}{V_d} = -0.375$, $\Delta I_{p-p} = 0.625 \text{ A}$ as in part b



7-5

The Conditions are identical to that in Problem 7-4a, where $\Delta I_{p-p} = 0.641 \text{ A}$. Therefore,

$$T_{em}(t) = \underbrace{\bar{T}_{em}}_{\text{average}} + T_{em,\,ripple}$$

where $\bar{T}_{em} = T_L = k_T I_a = 3 \text{ Nm}$

and $T_{em,\,ripple} (= k_T i_{a,\,ripple})$ has the same waveform as $i_{a,\,ripple}$. In the mechanical system,

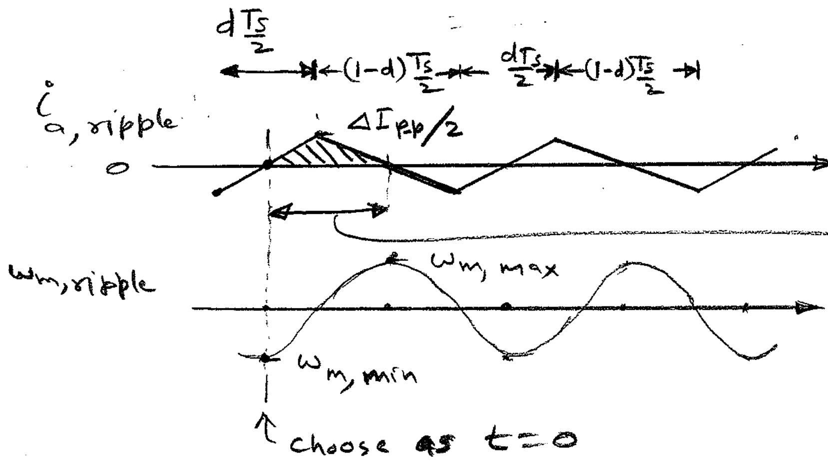
$$J_{eq} \frac{d\omega_m}{dt} = (T_{em} - T_L)$$ where $J_{eq} = J_m + J_L$
$$= 0.02 + 0.04$$
$$= 0.06 \text{ Nm}$$

$$= T_{em,\,ripple}$$

Therefore,

$$\frac{d\omega_m}{dt} = \frac{T_{em,ripple}}{J_{eq}} = \frac{k_T\, i_{a,ripple}}{J_{eq}}$$

$$= \frac{0.5}{0.06}\, i_{a,ripple} = 0.83\, i_{a,ripple}$$



$$= \frac{dT_s}{4} + (1-d)\frac{T_s}{4}$$

$$= \left(\frac{dT_s}{4} + \frac{T_s}{4} - \frac{dT_s}{4}\right) = \frac{T_s}{4}$$

$T_s = 40\,\mu s$

Since $\frac{d\omega_m}{dt} = 8.33\, i_{a,ripple}$, during $0 \le t \le$

$$\omega_{m,max} = \omega_{m,min} + 8.33 \int_0^{T_s/4} i_{a,ripple}$$

$$\therefore \Delta\omega_{m,p\text{-}p} = 8.33 \times \text{Shaded area}$$

$$= 8.33 \times \frac{1}{2}\left(\frac{\Delta I_{p\text{-}p}}{2}\right)\left(\frac{T_s}{4}\right)$$

$$= \frac{8.33}{16}\Delta I_{p\text{-}p}\, T_s = \frac{8.33}{16} \times 0.641 \times 40\times 10^{-6}$$

$$= 13 \times 10^{-6} \text{ rad/s} \approx 0$$

Notice that compared to the average speed of approximately 157 rad/s, the ripple in speed is negligible.

7-6

$\Delta I_{p-p} = 0.641\,A$

$I_{a\,(avg)} = 6.0$

Approach:

To get the exact answer will require that we find the rms value of i_a. This can be quite tedius. Therefore, we can approximate the ripple component by a sine wave which has the same peak-peak value as in the waveform shown above. Therefore,

$$i_a(t) \simeq I_{a,avg} + \underbrace{\frac{\Delta I_{p-p}}{2}}_{\text{amplitude}} \sin \omega_s t$$

amplitude $\Rightarrow \left(\frac{\Delta I_{p-p}}{2}\frac{1}{\sqrt{2}}\right)$ rms value

where $\omega_s = 2\pi f_s$.

$$\therefore\ I_{a,rms} \simeq \sqrt{I_{a,avg}^2 + \left(\frac{\Delta I_{p-p}}{2}\frac{1}{\sqrt{2}}\right)^2}$$

$$= 6.0043\ A$$

$$\therefore\ P_R = R_a I_{a,rms}^2 = 12.62\ W$$

$$P_R\,(\text{dc source}) = R_a \times I_{a,avg}^2 = 12.6\ W$$

$$\therefore\ \%\Delta P_{R,\text{additional}} = \frac{P_R - P_R\,(\text{dc source})}{P_R\,(\text{dc source})} \times 100 \simeq 0.2\%$$

The additional loss is negligibly small.

7-7

$$\omega_{m_1} = 1{,}500 \text{ rpm} = 157.08 \ \frac{\text{rad}}{s}$$

$$\omega_{m_2} = 750 \text{ rpm} = 78.54 \ \frac{\text{rad}}{s}$$

$$J_{eq} = J_m + J_L = 0.02 + 0.04 = 0.06 \ \text{kg}\cdot\text{m}^2$$

$$\therefore \ \Delta E_{inertia} = \frac{1}{2} J_{eq} \left[\omega_{m_1}^2 - \omega_{m_2}^2 \right] = \frac{1}{2} \times 0.06 \left[157.08^2 - 78.54^2 \right]$$

$$= 555.17 \ J$$

$$T_{em,\,braking} = k_T |I_a| = 0.5 \times 10 = 5 \ Nm$$

$$\frac{d\omega_m}{dt} = -\frac{T_{em}}{J_{eq}}$$

$$\therefore \ \omega_m(t) = \omega_{m_1} - \frac{T_{em}}{J_{eq}} t$$

$$\omega_{m_2} = \omega_{m_1} - \frac{T_{em}}{J_{eq}} \Delta t$$

or $$\Delta t = (\omega_{m_1} - \omega_{m_2}) \frac{J_{eq}}{T_{em}} = (157.08 - 78.54) \frac{0.06}{5}$$

$$= 78.54 \times \frac{0.06}{5} = 0.942 \ s$$

$$\therefore \ E_{loss} = R_a I_a^2 \Delta t = 0.35 \times 10^2 \times 0.942$$

$$= 32.97 \ J$$

$$\therefore \Delta E_{recovered} = \Delta E_{inertia} - E_{loss}$$

$$= 555.17 - 32.97$$

$$\simeq 522 \ J$$

7-8 To bring the motor to a steady state as quickly as possible, the maximum torque should be developed by the motor. $\therefore$ $T_{em} = k_T I_{a,max} = 0.5 \times 15 = 7.5$ Nm

$$T_{acc} = T_{em} - T_L = 7.5 - 2 = 5.5 \text{ Nm}$$

$$J_{eq} \frac{d\omega_m}{dt} = T_{acc}$$

or $$\omega_m(t) = \frac{T_{acc}}{J_{eq}} t \quad , \quad \omega_m(0) = 0$$

Therefore, t_1 when $\omega_m = 300$ rad/s can be calculated as

$$t_1 = \frac{300 \text{ rad/s} \cdot J_{eq}}{T_{acc}} = 300 \times \frac{0.06}{5.5}$$

$$= 3.273 \text{ s}$$

$0 \le t \le t_1$ $$\omega_m(t) = \frac{T_{acc}}{J_{eq}} t$$

$$= 91.667\, t$$

$$e_a(t) = k_E \omega_m(t) = 0.5 \times 91.667\, t$$
$$= 45.834\, t \text{ V}$$

$$R_a I_a = 0.35 \times 15 = 5.25 \text{ V}$$

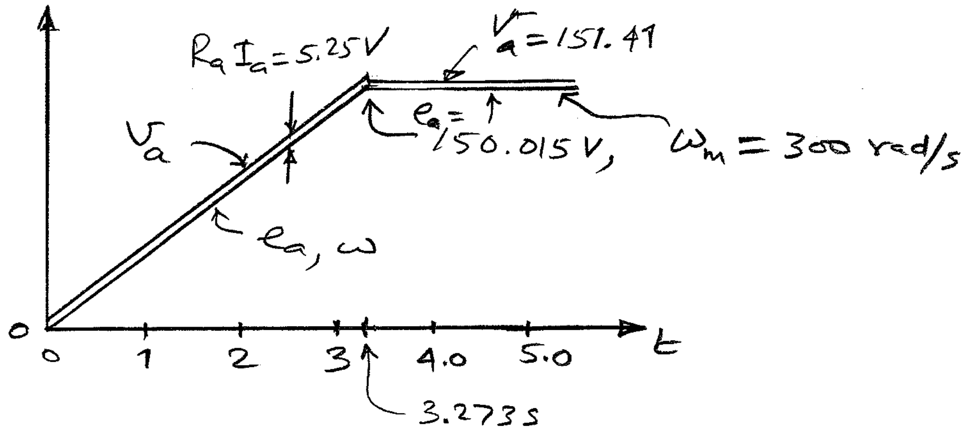
$\therefore$ $V_a(t) = (45.834t + 5.25)$ V $\quad 0 \le t \le t_1$

$$= 150.015 + 5.25 = 155.265 \text{ V}$$

Beyond t_1, $T_{em} = T_L = 2$ Nm $\quad \therefore$ $I_a = \frac{2}{0.5} = 4$ A

$\therefore$ $R_a I_a = 0.35 \times 4 = 1.4$ V

and, $V_a = 150.015 + 1.4 = 151.41$ V



7-9

$$\omega_{m1} = 300 \text{ rad/s} \quad \text{at } t=0$$

$$\omega_{m2} = -100 \text{ rad/s} \quad \text{at } t=4s \text{ , linearly}$$

$$J_{eq} = 0.02 + 0.04 = 0.06 \text{ kg}\cdot\text{m}^2$$

$$T_{em} = J_{eq}\frac{\Delta\omega_m}{\Delta t} = 0.06\,\frac{-100-300}{4s}$$

$$= -6.0 \text{ Nm}$$

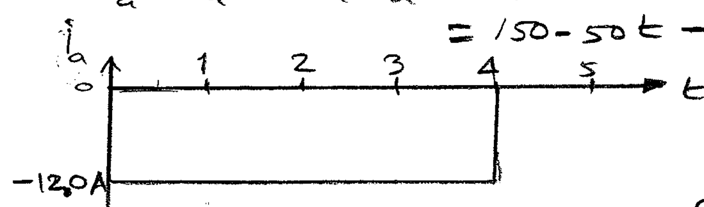
$$\therefore\ I_a = \frac{T_{em}}{k_T} = -12.0 \text{ A}$$

$$\omega_m(t) = 300 - \frac{400}{4}t = 300 - 100t$$

$$\therefore\ e_a = k_E\,\omega_m = (150 - 50t)\text{ V}$$

$\therefore$ $0 < t < 4s$ $\quad V_a = e_a + R_a I_a = (150 - 50t) - 0.35\times 12.0$

$$= 150 - 50t - 4.2\text{ V}$$



at $t=0$, $V_a = 145.8V$

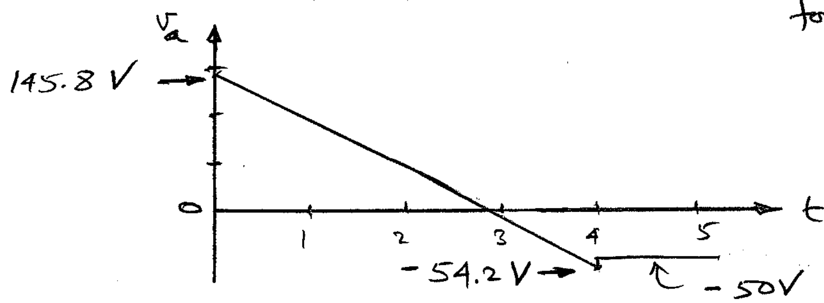
at $t=4s$

$V_a = -54.2\,V$

$e_a = -50V$

for $t > 4s$

$i_a = 0 \quad \therefore\ v_a = e_a = -50V$



7-10

$T_L = 0$ at $\omega_m = 0$

$= 4\ Nm$ at $300\ rad/s$

$\therefore T_L = \left(\frac{4}{300}\right)\omega_m = 1.33\times10^{-2}\,\omega_m$

$J_m = 0.02\ kg\cdot m^2$, $J_L = 0.04\ kg\cdot m^2$

$J_{eq} = 0.06\ kg\cdot m^2$

The maximum current of 15 A should be applied.

$$\frac{d\omega_m}{dt} = \frac{T_{em} - T_L}{J_{eq}} = \frac{0.5\times15 - 1.33\times10^{-2}\,\omega_m}{0.06}$$

$$= 125 - 0.222\,\omega_m$$

or, $$\frac{d\omega_m}{dt} + 0.222\,\omega_m = 125.0$$

$$\therefore \quad \omega_m(t) = 563.06\left(1 - e^{-0.222t}\right)$$

at $t = t_1$, $\omega_m(t_1) = 300\ rad/s$

$$\therefore \quad 300 = 563.06\left(1 - e^{-0.222t_1}\right)$$

$$e^{-0.222t_1} = 0.467$$

$$-0.222t_1 \underbrace{\ln(e)}_{1} = \ln(0.467) = -0.761$$

$$\therefore t_1 = 3.43\ s$$

$0 \le t \le 3.43s$

$$e_a = k_E\omega_m = 281.53\left(1 - e^{-0.222t}\right)$$

$$V_a = e_a + R_a \underset{(=15A)}{I_a} = 281.53\left(1 - e^{-0.222t}\right) + (0.35\times15)$$

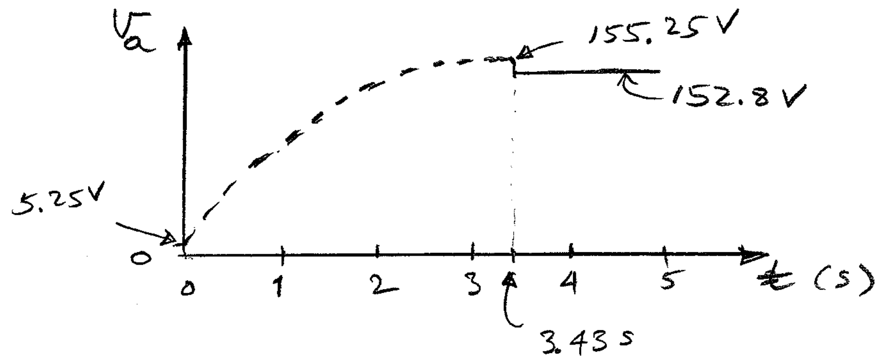
at $t = 3.43s \Rightarrow V_a = 150 + 5.25 = 155.25\ V$

$t > 3.43s$ $\omega_m = 300\ rad/s$ $T_L = 4\ Nm$, $I_a = \frac{T_L}{0.5} = 8.0A$

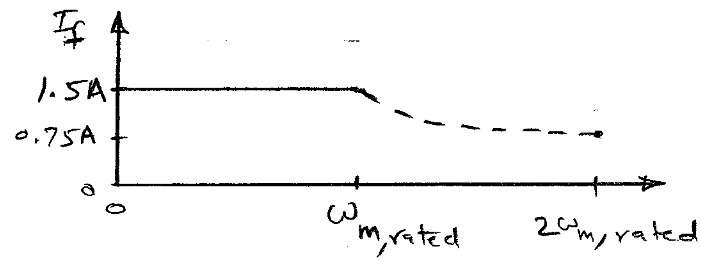
$$e_a = k_E \omega_m = 150\ V$$

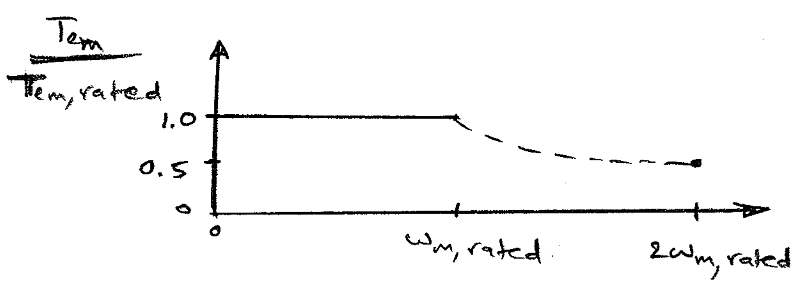
$$R_a I_a = 0.35 \times 8.0 = 2.8\ V$$

$$\therefore V_a = 150 + 2.8 = 152.8\ V$$

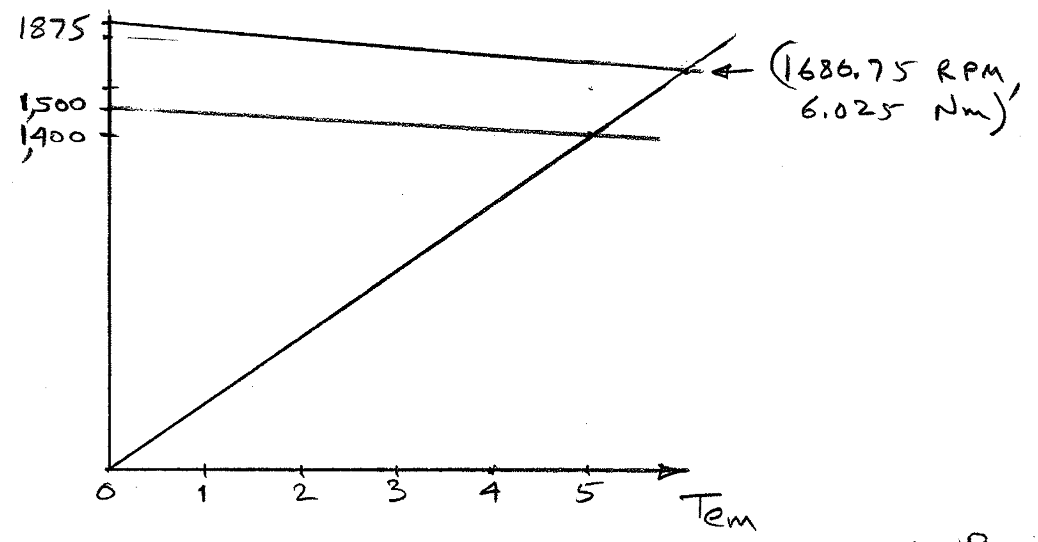


7-11 $\omega_{m,rated} = 2000\ rpm = 209.44\ \frac{rad}{s}$





7-12



at B_f = rated

$\Delta\omega_m = 1500 - 1400$

$= 100$ rpm

$$\omega_m\Big|_{\text{no-load}}^{B_f=0.8\,\text{rated}} = \frac{1500}{0.8} = 1875 \text{ rpm}$$

$$V_a = k_e B_f \omega_m - R_a I_a$$

$$I_a = \frac{T_{em}}{k_t B_f}$$

$$\therefore \quad V_a = k_e B_f \omega_m - R_a \frac{T_{em}}{k_t B_f}$$

$$\omega_m = \underbrace{\frac{V_a}{k_e B_f}}_{1875\text{ rpm}} - \underbrace{R_a \frac{T_{em}}{k_t B_f \, k_e B_f}}$$

$$\Delta\omega_m\Big|_{B_f=0.8\,\text{rated}} = \Delta\omega_m\Big|_{B_f=\text{rated}} \frac{1}{B_f^2}$$

$$= \frac{100}{0.8^2} = 156.25$$

$$\therefore \omega_m\Big|_{T_{em}=5\text{ Nm}}^{B_f=0.8\,\text{rated}} = 1875 - 156.25 = 1718.75 \text{ rpm}$$

Motor torque-speed ch. at $B_f = 0.8$ rated $\qquad \omega_m = 1875 - \frac{156.25}{5} \cdot T_{em} \qquad (1)$

Load torque-speed ch $\qquad T_{em} = \frac{5}{1400}\,\omega_m \qquad (2)$

Substituting (2) into (1)

$$\omega_m = 1875 - \frac{15625}{\cancel{5}} \frac{\cancel{5}}{1400} \omega_m$$

$$\therefore \omega_m \left(1 + \frac{156.25}{1400}\right) = 1875$$

or $\omega_m = 1686.75$ rpm

$$T_{em} = T_L = \frac{5}{1400} \times 1686.75 = 6.024 \text{ Nm}$$

7-13

$$e_{ph-ph} = k_E \omega_m = 0.75 \times 100 = 75 \text{ V}$$

$\therefore$ The flat portion of the phase emf waveforms have a value of $\frac{e_{ph-ph}}{2} = 37.5$ V in fig. 7-23.

$$T_{em} = k_T I$$

$$\therefore I = \frac{T_{em}}{k_T} = \frac{6.0}{0.75} = 8.0$$

Therefore, in fig. 7-24c, the rectangular current pulses have an amplitude of 8.0 A.

7-14

Regenerative braking can be achieved by making the motor currents to negative of the waveforms shown in Fig. 7-24C for the three phase currents.

Chapter 8

8-1

$$G_{OL} = \frac{k}{1+s/\omega_p}$$

$$G_{CL} = \frac{G_{OL}}{1+G_{OL}} = \frac{\frac{k}{1+s/\omega_p}}{1+\frac{k}{1+s/\omega_p}} = \frac{k}{(k+1)+s/\omega_p}$$

$$= \frac{k}{k+1}\,\frac{1}{1+\frac{s}{(k+1)\omega_p}}$$

for large values of $k \gg 1$, $\frac{k}{k+1} \simeq 1$

$$G_{CL} \simeq \frac{1}{1+\frac{s}{(k+1)\omega_p}}$$

Therefore, the bandwidth of the closed-loop transfer function is $(k+1)\omega_p$, which depends on both k and ω_p.

8-2

$$G(s) = \frac{k}{1+s/\omega_p}$$

$$H(s) = k_{fb} \ (< 1)$$

$$\therefore\ G_{CL}(s) = \frac{G(s)}{1+GH(s)} = \frac{\frac{k}{1+s/\omega_p}}{1+\frac{k k_{fb}}{1+s/\omega_p}}$$

$$= \frac{k}{1+k k_{fb}+s/\omega_p}$$

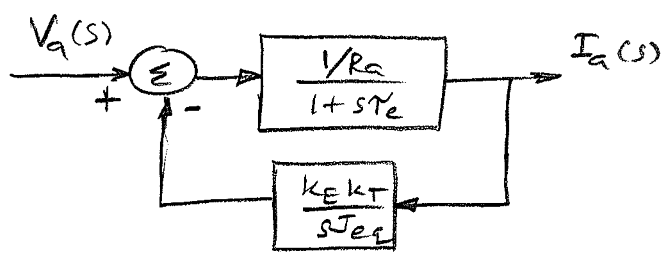
$$= \frac{k}{1+k k_{fb}}\,\frac{1}{1+\frac{s}{(1+k k_{fb})\omega_p}}$$

In practice, $k >> 1$, therefore (although $k_{fb} < 1$), $k k_{fb} >> 1$ and $1 + k k_{fb} \simeq k k_{fb}$.

$$\therefore \quad G_{CL}(s) \simeq \frac{k}{k k_{fb}} \quad \frac{1}{1 + \frac{s}{k k_{fb} \omega_p}}$$

The bandwidth of the closed-loop transfer function is $\sim k k_{fb} \omega_p$.

8-3 Including the effect of the back-emf, from fig. 8-9 b,



$$\therefore \quad \frac{I_a(s)}{V_a(s)} = \frac{\frac{1/R_a}{1+s\tau_e}}{1 + \frac{1/R_a}{1+s\tau_e} \cdot \frac{k_E k_T}{s J_{eq}}}$$

$$= \frac{\frac{1/R_a}{\cancel{1+s\tau_e}}}{\frac{(1+s\tau_e)(sJ_{eq}) + \frac{1}{R_a} k_E k_T}{\cancel{(1+s\tau_e)}\, s J_{eq}}}$$

$$= \frac{s J_{eq}}{R_a} \quad \frac{1}{s^2 \tau_e J_{eq} + s J_{eq} + \frac{k_E k_T}{R_a}}$$

$$= \frac{s \cancel{J_{eq}}}{R_a \tau_e \cancel{J_{eq}} \left(s^2 + \frac{1}{\tau_e} s + \frac{k_E k_T}{R_a \tau_e J_{eq}}\right)}$$

Substituting the numerical values $\left(\tau_e = \frac{L_a}{R_a} = \frac{5.2\,mH}{2\,\Omega} = 2.6\times10^{-3}\,s\right)$

$$\frac{I_a}{V_a}(s) = \frac{s}{2\times2.6\times10^{-3}\left(s^2 + \frac{1}{2.6\times10^{-3}}\,s + \frac{0.1\times0.1}{2\times2.6\times10^{-3}\times152\times10^{-6}}\right)}$$

$$= 192.31\,\frac{s}{(s^2 + 384.62\,s + 12651.8)}$$

If $s^2 + 384.62s + 12651.8 = 0$

$$s = \frac{-384.62 \pm \sqrt{384.62^2 - 4\times12651.8}}{2}$$

$$= \frac{-384.62 \pm 311.97}{2} = -348.3,\ -36.3$$

$$\therefore\quad s^2 + 384.62s + 12651.8 \simeq (s+348.3)(s+36.3)$$

$$\therefore\quad \frac{I_a}{V_a}(s) = \frac{192.31}{348.3\times36.3}\,\frac{s}{(1+s/348.3)(1+s/36.3)}$$

$$= 0.015\,\frac{s}{\left(1+\frac{s}{348.3}\right)\left(1+\frac{s}{36.3}\right)}$$

Near the cross-over frequency of 1 kHz ($\omega_c = 2\pi\,kHz$), $\left(1+\frac{s}{36.3}\right) \approx \frac{s}{36.3}$

$$\therefore\quad \frac{I_a}{V_a}(s) \simeq 0.015\,\frac{\cancel{s}}{\left(1+\frac{s}{348.3}\right)\left(\frac{\cancel{s}}{36.3}\right)} \simeq \frac{0.545}{1+s/348.3} \qquad (1)$$

In Example 18-2, where the effect of back-emf was neglected

$$\frac{I_a(s)}{V_a(s)} = \frac{1/R_a}{1+s\tau_e} = \frac{0.5}{1+s/384.62} \qquad (2)$$

Note that the transfer functions in Eq. (1) and Eq. (2) are not very different, which shows that neglecting the effect of back-emf in Example 8-2 was justified.

From Eq. 1, it is clear that the factor 0.995 is very close to unity and its effect on the new design will be negligible.

From Eq. 2

$$\angle \frac{k_{i\Omega} k_T}{J_{eq}} \frac{1 + s/(k_{i\Omega}/k_{p\Omega})}{s^2} \Bigg|_{s=j\omega_{c\Omega}} = -180^\circ + (\phi_{PM,\Omega} + 5.7^\circ)$$

In the design procedure of Example 8-3, $(\phi_{PM,\Omega} + 5.7^\circ)$ was equal to 60°. Therefore, the design was, in fact, for a $\phi_{PM,\Omega}$ of 54.3°

If the design is carried out by including 5.7° due to the torque (current) loop, the new system will be slightly less oscillatory due to a slightly higher phase margin (by 5.7°) compared to the design in Example 8-3.

8-4

In the torque (current) loop, from Eq. 8-19

$$G_{I,OL} = \frac{k_{iI}\, k_{PWM} / R_a}{s}$$

From the parameters calculated in Example 8-2,

$$G_{I,OL} = \frac{1050 \times 12 / 2.0}{s} = \frac{6300}{s}$$

$$\therefore \quad G_{I,CL} = \frac{G_{I,OL}}{1+G_{I,OL}} = \frac{\frac{6300}{s}}{1+\frac{6300}{s}} = \frac{6300}{s+6300}$$

$$= \frac{1}{1+s/6300}$$

In Example 8-3, the crossover freq. of the speed loop is 100 Hz or 628 rad/s.

$$\therefore \quad \left| G_{I,CL} \right|_{s=j628} = 0.995$$ (assumed to be unity in Example 8-3)

and $$\angle G_{I,CL} \big|_{s=j628} = -5.7^\circ$$ (assumed to be zero in Example 8-3

$\therefore$ In Eq. 8-23

$$\left| \frac{k_{i\Omega}\, k_T}{J_{eq}} \frac{1+ s/(k_{i\Omega}/k_{p\Omega})}{s^2} \right|_{s=j\omega_{c\Omega}} \times 0.995 = 1 \qquad (1)$$

Similarly,

$$\angle \frac{k_{i\Omega}\, k_T}{J_{eq}} \frac{1+ s/(k_{i\Omega}/k_{p\Omega})}{s^2} \Bigg|_{s=j\omega_{c\Omega}} - 5.7^\circ = -180^\circ + \phi_{PM,\Omega} \qquad (2)$$

8-5

From Eq. 8-22

$$G_{\Omega,OL} = k_1 \frac{1+s/\omega_z}{s^2}$$

where $k_1 = \frac{k_{i\Omega} k_T}{J_{eq}} = \frac{299.7 \times 0.1}{152 \times 10^{-6}} = 0.197 \times 10^6 \simeq 2 \times 10^5$

and $\omega_z = k_{i\Omega}/k_{p\Omega} = \frac{299.7}{0.827} = 362.4$ rad/s

$$\therefore \quad G_{\Omega,OL} = 2 \times 10^5 \frac{1+s/362.4}{s^2} = 551.9 \frac{362.4+s}{s^2}$$

$$\therefore \quad G_{\Omega,CL} = \frac{G_{\Omega,OL}}{1+G_{\Omega,OL}} = \frac{551.9 \frac{362.4+s}{s^2}}{1+551.9 \times \frac{362.4+s}{s^2}}$$

$$= \frac{551.9 \times (362.4+s)}{s^2+551.9s+2\times 10^5}$$

$f_{co} = 10$ Hz, $\omega_{co} = 62.8 \frac{rad}{s}$

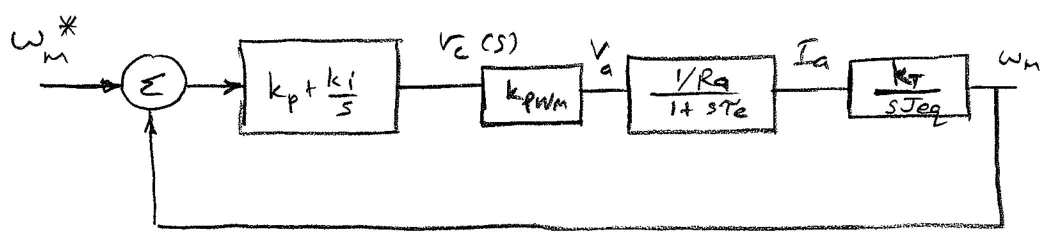
$$\therefore \quad G_{\Omega,CL}\Big|_{s=j\omega_{co}} = 1.02 \angle -0.172^\circ$$

Note the $G_{\Omega,CL}(s)$ is so close to unity at the position-loop crossover frequency of 62.8 rad/s that it is perfectly justified to assume it to be unity.

8-6

Large disturbances required large change in voltage and current. If the initial steady state operating point is close to the voltage and/or current limit, it is not possible to apply a large change in voltage and/or current. As a consequence, the system response may be sluggish and may also oscillate.

8-10



$$G_{OL}(s) = \frac{k_i}{s}\left[1+\frac{s}{k_i/k_p}\right]\frac{k_{PWM}\,k_T}{R_a J_{eq}}\;\frac{1}{(1+s\tau_e)s}$$

To calculate the two unknowns k_p and k_i, note that

$$\left|G_{OL}(s)\right|_{s=j\,628\text{ rad/s}} = 1 \quad \text{and} \quad \angle G_{OL}(s)\Big|_{s=j628} = -180^\circ + \underbrace{\phi_{PM}}_{(=60^\circ)}$$

$$= -120^\circ$$

$$\therefore \quad \frac{k_i \, k_{pwm} \, k_T}{R_a \, J_{eq} \, (628)^2} \sqrt{1+\left(\frac{628}{k_i/k_p}\right)^2} \; \underbrace{0.522}_{= \left|\frac{1}{(1+s\tau_e)}\right|} = 1$$

$$\text{or} \quad k_i \sqrt{1+\left(\frac{628}{k_i/k_p}\right)^2} = \frac{1}{0.522} \times \frac{R_a \, J_{eq} \, (628)^2}{k_{pwm} \, k_T}$$

$$= 191.4$$

$$\text{or} \quad k_i^2\left[1+\left(\frac{628}{k_i/k_p}\right)^2\right] = 191.4^2$$

$$\text{or} \quad k_i^2 + 628^2 \, k_p^2 = 191.4^2 \qquad (1)$$

And, at $s = j628$

$$\underbrace{-180^\circ}_{\text{due to } \frac{1}{s^2}} + \angle\left(1+s\frac{k_p}{k_i}\right) - \angle 1+s\tau_e = -120^\circ$$

$$\text{or} \quad \tan^{-1}\left(\frac{628}{k_i}k_p\right) - \tan^{-1}(628\tau_e) = 60^\circ$$

$$\text{or,} \quad \tan^{-1}\frac{628 \, k_p}{k_i} = 60^\circ + 58.51^\circ = 118.51^\circ$$

$$\therefore \quad 628\frac{k_p}{k_i} = \tan 118.51^\circ = -1.841 \qquad (2)$$

Since both kp and ki have positive values, it is clearly not possible to satisfy Eq. 2. Therefore, without an inner torque (current) loop, it is not possible to achieve a cross-over frequency of 1 kHz and a phase margin of 45°.

Therefore, we will pick 0.5 kHz as the crossover frequency and 45° as the phase margin.

To recalculate Eq. (1)

$$\text{at} \quad \omega_c = \frac{628}{2} = 314 \text{ rad/s}$$

$$\left| \frac{1}{1+s\tau_e} \right|_{s=j314 \text{ rad/s}} = 0.775$$

$$\therefore \quad k_i \sqrt{1+\left(\frac{314\, k_p}{k_i}\right)^2} = \frac{1}{0.775} \times \frac{R_a J_{eq} (314)^2}{k_{pwm}\, k_T}$$

$$= 32.24$$

$$\therefore \quad k_i^2 \left[1 + \left(314 \frac{k_p}{k_i}\right)^2 \right] = 32.24^2 \qquad (1')$$

and,

to recalculate (2)

$$\tan^{-1}\left(\frac{314\, k_p}{k_i}\right) = 45^\circ + \tan^{-1}(314\tau_e)$$

$$= 45^\circ + 39.23^\circ = 84.23^\circ$$

$$\therefore \quad \frac{314\, k_p}{k_i} = 9.89 \quad \text{or} \quad \frac{k_p}{k_i} = 0.032 \qquad (2')$$

Substituting for $314 \frac{k_p}{k_i}$ in Eq. (1'),

$$k_i^2 (1 + 9.89^2) = 32.24^2$$

$\therefore \; k_i = 3.24$ and from Eq. (2'),

$$k_p = 0.032 \times k_i = 0.1$$

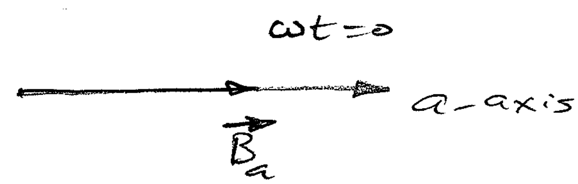
Chapter 9

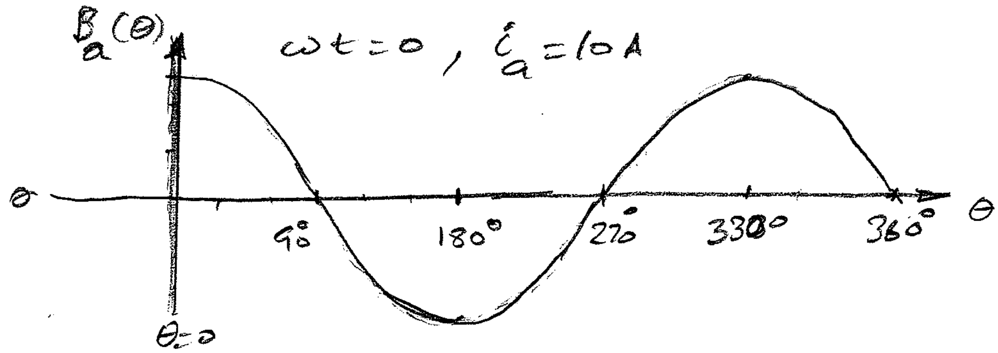
9-1 $\quad i_a = 10 \sin \omega t$, N_s and l_g are given.

(a) $\omega t = 0$; $\quad i_a = 0 \quad \therefore \quad \vec{B}_a = 0$

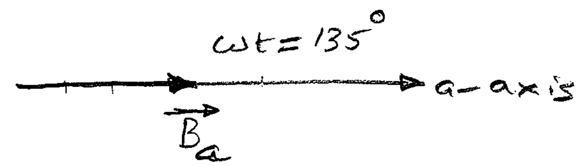
$B_a(\theta) = 0$ everywhere.

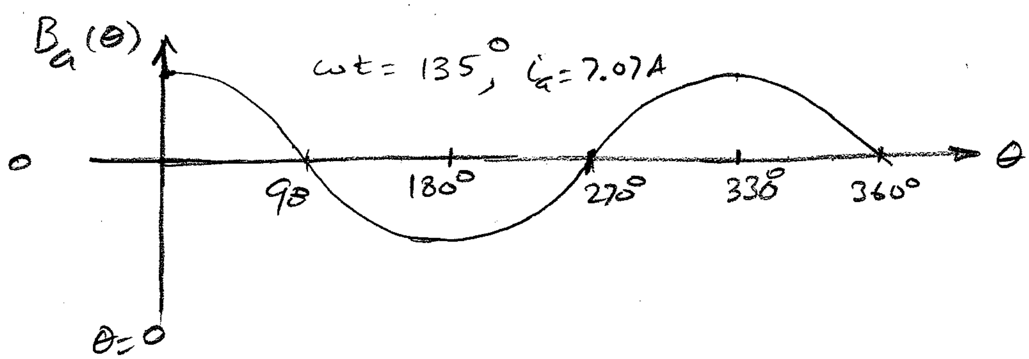
(b) $\omega t = 90^\circ$; $\quad i_a = 10$ A $\quad \therefore \quad \vec{B}_a = \mu_0 \dfrac{10 N_s}{2 l_g} \angle 0^\circ$





(C) $\omega t = 135^\circ$; $\quad i_a = 7.07$ A $\quad \therefore \quad \vec{B}_a = \mu_0 \dfrac{7.07 N_s}{2 l_g} \angle 0^\circ$

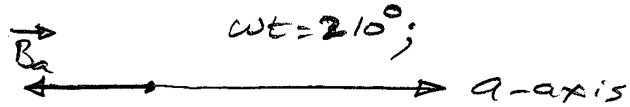


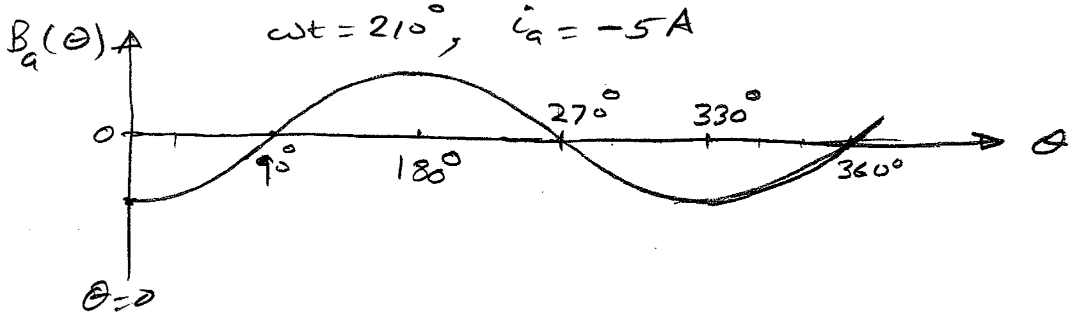


(d) $\omega t = 210^\circ$, $i_a = -5\,A$ $\therefore$ $\vec{B}_a = -\dfrac{\mu_o 5 N_s}{2 l_g} \angle 0^\circ$

$= \mu_o \dfrac{5 N_s}{2 l_g} \angle 180^\circ$

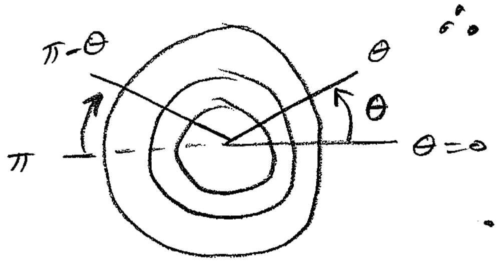
$\omega t = 210^\circ$;





9-2

By symmetry, at $\pi - \theta$, $H_a(\pi - \theta) = -H_a(\theta)$



$\therefore$ Applying Ampere's Law:

$$H_a(\theta) l_g - H_a(\pi - \theta) l_g = \int_{\theta}^{\pi-\theta} i_a\, n_s(\xi) \cdot d\xi$$

$$\therefore\ 2 H_a(\theta) l_g = \int_{\theta}^{\pi-\theta} i_a (N_s/2) \sin(\xi) \cdot d\xi$$

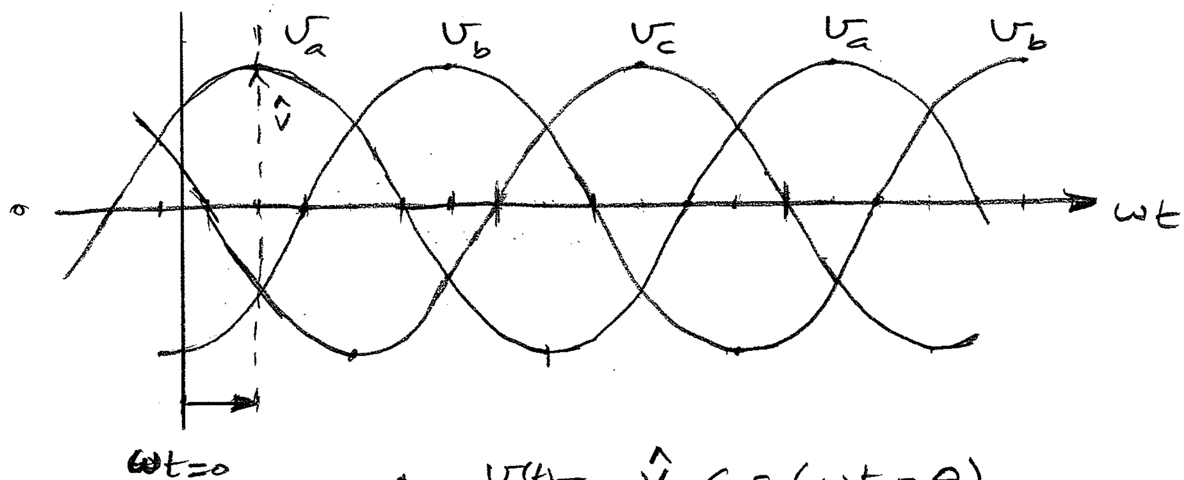
$$\therefore\ H_a(\theta) = \frac{(N_s/2)\, i_a}{2 l_g} \int_{\theta}^{\pi-\theta} \sin \xi \cdot d\xi$$

$$= \frac{(N_s/2)\, i_a}{2 l_g} \cos \xi \Big|_{\pi-\theta}^{\theta} = \frac{N_s}{2 l_g}\, i_a \cos\theta$$

Note that the expression is exactly the same as in Eq. 9-7.

9-3

In time domain: a-b-c sequence



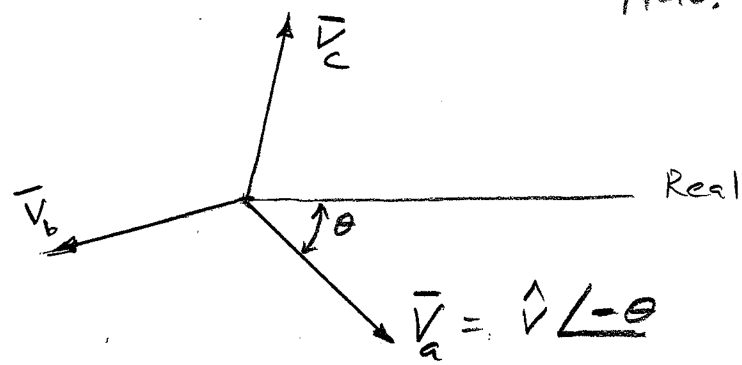
$$\therefore \quad v_a(t) = \hat{V} \cos(\omega t - \theta)$$

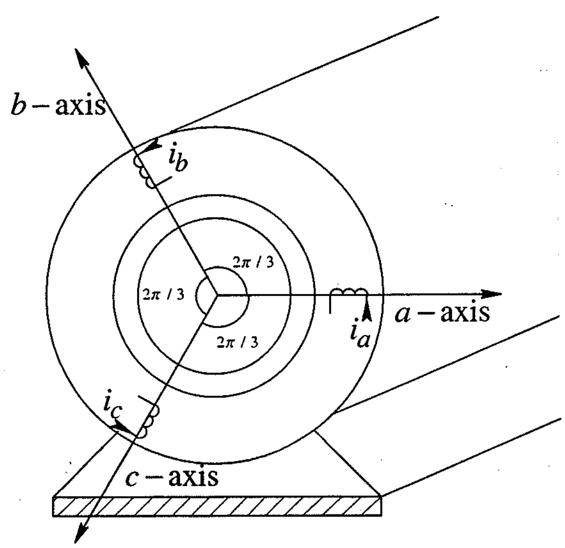
$$v_b(t) = \hat{V} \cos(\omega t - \theta - 120^\circ)$$

$$v_c(t) = \hat{V} \cos(\omega t - \theta - 240^\circ)$$

In Phasor Domain

Note: Phase-b voltage lags behind phase-a voltage by 120°.



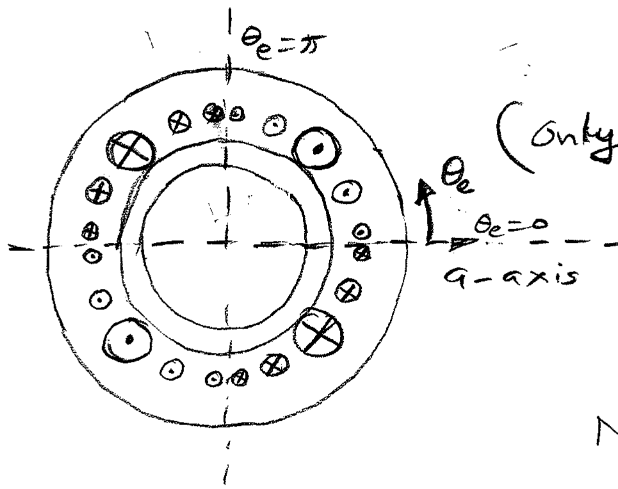


In the airgap, the a-b-c sequence causes the flux-density distribution to rotate counter-clockwise. Therefore, the induced voltage in phase-a peaks first, then $\omega t (= 120^\circ)$ later in phase-b (which is placed 120° ahead physically, compared to phase-a).

9-4

Derive that $n_s(\theta_e) = \frac{N_s}{2} \sin\theta_e$ Eq. 9-11

where, $\theta_e = \frac{p}{2}\theta$



4-pole machine (Only phase-a is shown)

$$n_s(\theta_e) = \hat{n}_s \sin\theta_e \quad (1)$$

where $\hat{n}_s = ?$ (to be calculated)

N_s = total number of turns per phase

$2N_s$ = total number of conductors per phase

For a p-pole machine, between $\theta = 0$ and $\theta = \frac{2\pi}{p}$ rad, $\left(\frac{1}{p}\right)^{th}$ of the total conductors are located, that is equal to $\frac{2N_s}{p} = \frac{N_s}{(p/2)}$ conductors.

$$\therefore \int_0^{2\pi/p} \hat{n}_s \sin\left(\tfrac{p}{2}\theta\right) d\theta = \frac{2}{p}\hat{n}_s \cos\left(\tfrac{p}{2}\theta\right)\Big|_{2\pi/p}^{0} = \frac{2}{p}\hat{n}_s \underbrace{\left[\cos(0) - \cos\left(\tfrac{p}{2}\cdot\tfrac{2\pi}{p}\right)\right]}_{(=2)} = \frac{4}{p}\hat{n}_s \quad (2)$$

The integral in Eq. (2) also equals $\frac{N_s}{(p/2)}$.

$$\therefore \frac{4}{p}\hat{n}_s = \frac{N_s}{p/2}$$

or, $\hat{n}_s = \frac{N_s}{2}$ (this is indendent of the number of poles) (3)

Substituting for $\hat{n}_s$ from Eq. (3) into Eq. (1),

$$n_s(\theta_e) = \frac{N_s}{2}\sin\theta_e$$

which confirms Eq. 9-11 for any $p \geq 2$.

9-5

In a p-pole machine, let's define $\boxed{N_{sp} = \frac{N_s}{p}}$

$H_a(\theta_e) = -H_a(\theta_e + \pi)$; (see figure below for $p=4$)

Using this symmetry (it exists in general for a p-pole machine)

$$l_g H_a(\theta_e) - l_g H_a(\theta_e + \pi) = i_a \int_{\theta}^{\theta + \pi/(p/2)} \frac{N_s}{2} \sin\left(\frac{p}{2}\xi\right) \cdot d\xi$$

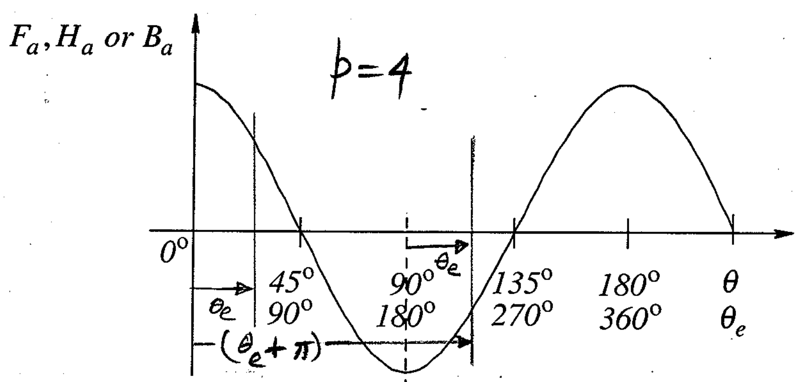
$$\therefore \quad 2 H_a(\theta_e) = \frac{i_a}{l_g} \frac{N_s}{2} \frac{2}{p} \cos\left(\frac{p}{2}\xi\right)\Big|_{\theta + \frac{\pi}{p/2}}^{\theta}$$

or,

$$H_a(\theta_e) = \frac{i_a}{l_g}\left(\frac{N_s}{2p}\right) \underbrace{\left[\cos(\underbrace{\tfrac{p}{2}\theta}_{\theta_e}) - \cos(\underbrace{\tfrac{p}{2}\theta + \pi}_{\theta_e + \pi})\right]}_{2\cos\theta_e}$$

$$\therefore \quad H_a(\theta_e) = \left(\frac{N_s}{p}\right) \frac{1}{l_g} i_a \cos\theta_e = \frac{N_{sp}}{l_g} i_a \cos\theta_e$$

$$B_a(\theta_e) = \mu_o \frac{N_{sp}}{l_g} i_a \cos\theta_e; \quad F_a(\theta_e) = N_{sp} i_a \cos\theta_e$$



9-6 $\quad N_s = 100$ turns

$$\vec{F}_s(t) = (N_s/2)\,\vec{i}_s(t)$$

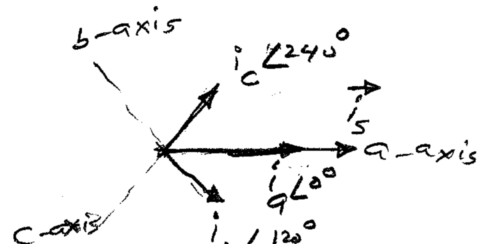
$$\vec{i}_s(t) = i_a(t) + i_b(t)\angle 120^\circ + i_c(t)\angle 240^\circ$$

(a) $\quad i_a = 10\text{ A}, \quad i_b = i_c = -5\text{ A}$

$$\therefore \quad \vec{i}_s = 10 + (-5)\angle 120^\circ + (-5)\angle 240^\circ$$

$$= 15\angle 0^\circ \text{ A}$$

$$\vec{F}_s = 750\angle 0^\circ \text{ A-turns}$$

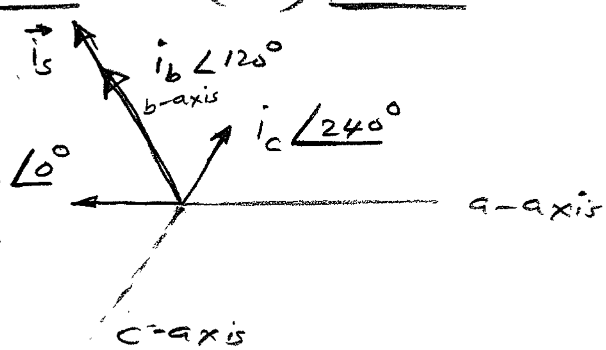


(b) $\quad i_a = -5\text{ A}, \quad i_b = 10\text{ A}, \quad i_c = -5\text{ A}$

$$\therefore \quad \vec{i}_s = (-5) + 10\angle 120^\circ + (-5)\angle 240^\circ$$

$$= 15\text{ A}\angle 120^\circ$$

$$\vec{F}_s = 750\angle 120^\circ \text{ A-turns}$$

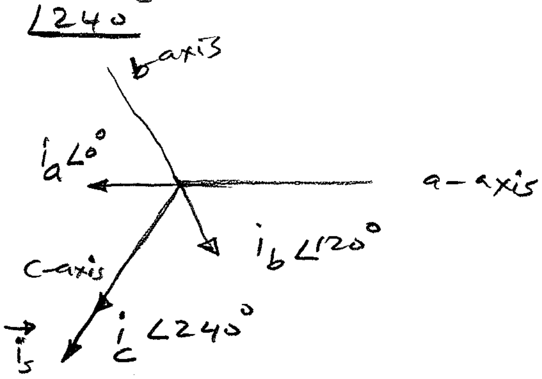


(c) $\quad i_a = -5\text{ A}, \quad i_b = -5\text{ A}, \quad i_c = 10\text{ A}$

$$\therefore \quad \vec{i}_s = (-5) + (-5)\angle 120^\circ + 10\angle 240^\circ$$

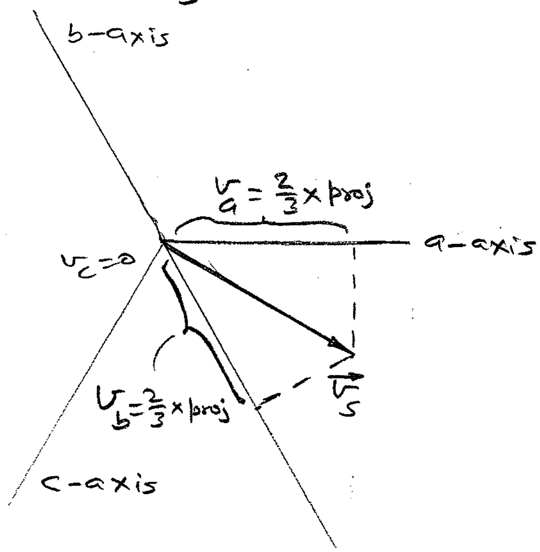
$$= 15\angle 240^\circ \text{ A}$$

$$\vec{F}_s = 750\angle 240^\circ \text{ A-turns}$$



9-7

$$\vec{v}_s = 150\angle{-30^\circ}\ V \qquad v_a,\ v_b,\ v_c = ?$$



$$v_a = \frac{2}{3}\hat{V}_s \cos(-30^\circ) = 86.6\ V$$

$$v_b = \frac{2}{3}\hat{V}_s \cos(-30^\circ - 120^\circ) = -86.6\ V$$

$$v_c = \frac{2}{3}\hat{V}_s \cos(-30^\circ - 240^\circ) = 0\ V$$

9-8 Eq. 9-26

$$\cos\omega t\angle{0^\circ} + \cos\left(\omega t - \frac{2\pi}{3}\right)\angle{120^\circ} + \cos\left(\omega t - \frac{4\pi}{3}\right)\angle{240^\circ}$$

$$= \cos\omega t + \cos\left(\omega t - \frac{2\pi}{3}\right)\left[\cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}\right] + \cos\left(\omega t - \frac{4\pi}{3}\right)\left[\cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}\right]$$

$$= \cos\omega t + \left[\cos\omega t \cdot \underbrace{\cos\left(\frac{2\pi}{3}\right)}_{-0.5} + \sin\omega t \cdot \underbrace{\sin\left(\frac{2\pi}{3}\right)}_{0.866}\right](-0.5 + j0.866) + \left[\cos\omega t \cdot \underbrace{\cos\left(\frac{4\pi}{3}\right)}_{-0.5} + \sin\omega t \cdot \underbrace{\sin\left(\frac{4\pi}{3}\right)}_{-0.866}\right](-0.5 - j0.866)$$

$$= \cos\omega t + \cos\omega t\left[(-0.5)(-0.5) + \cancel{(-0.5)(j0.866)} + (-0.5)(-0.5) + \cancel{(-0.5)(-j0.866)}\right] + \sin\omega t\left[\cancel{(0.866)(-0.5)} + (0.866)(j0.866) + \cancel{(-0.866)(-0.5)} + (-0.866)(-j0.866)\right]$$

$$= \cos\omega t + \frac{1}{2}\cos\omega t + j\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2}\sin\omega t$$

$$= \frac{3}{2}\cos\omega t + j\frac{3}{2}\sin\omega t = \frac{3}{2}(\cos\omega t + j\sin\omega t) = \frac{3}{2}\angle{\omega t}$$

9-9 $\quad \ell_g = 1.5$ mm, $\quad N_s = 100$ turns

$\hat{I}_m = 10$ A; peak of phase magnetizing currents

$i_a(t) = 10 \cos \omega t$ A

at $\omega t = 0$,

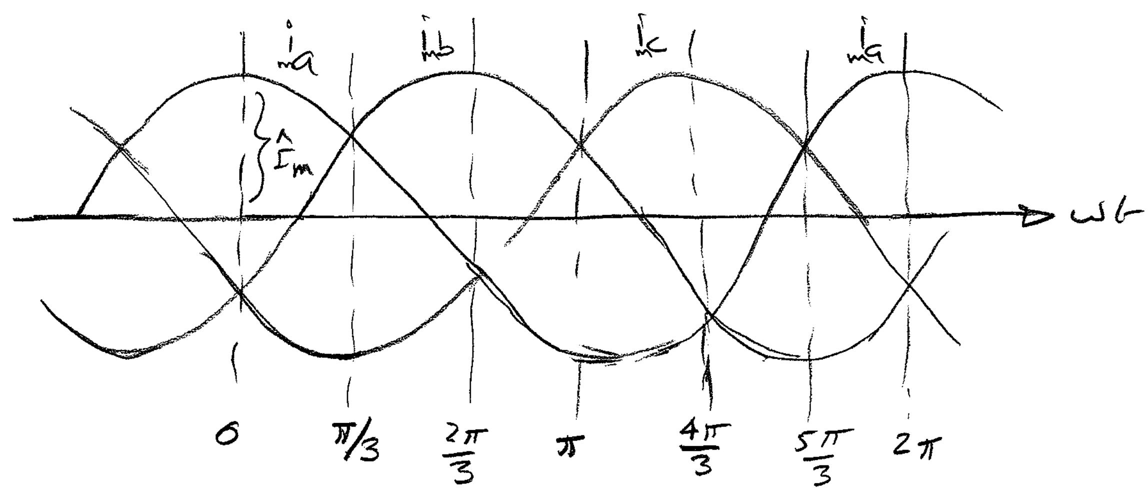
$$\vec{B}_s(\theta^\circ) = \frac{3}{2} \frac{N_s \hat{I}_m}{2\ell_g} \mu_o \angle 0^\circ = 0.628 \angle 0^\circ \text{ T}$$

$$\omega_{syn} = 60 \text{ rps} = 3{,}600 \text{ rpm}$$

9-10 $\quad p = 6$ poles

From Eq. 9-31, $\quad \omega_{syn} = \dfrac{\omega}{p/2} = \dfrac{60}{3}$ rps $= 20$ rps $= 1{,}200$ rpm

9-11



$\omega t = 0$: $\quad i_{ma} = \hat{I}_m$, $\quad i_{mb} = i_{mc} = -\hat{I}_m/2$

$\omega t = \frac{\pi}{3}$: $\quad i_{ma} = i_{mb} = \frac{\hat{I}_m}{2}$, $\quad i_{mc} = -\hat{I}_m$

$\omega t = \frac{2\pi}{3}$ $\quad i_{ma} = i_{mc} = -\frac{\hat{I}_m}{2}$, $\quad i_{mb} = \hat{I}_m$

$\omega t = \pi$ $\quad i_{ma} = -\hat{I}_m$, $\quad i_{mb} = i_{mc} = \hat{I}_m/2$

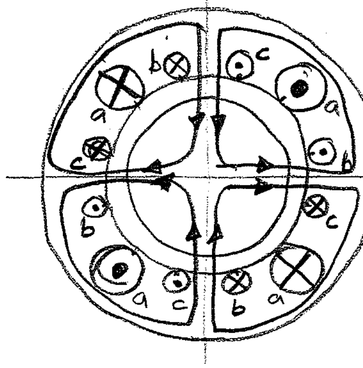
$\omega t = 4\pi/3$:

$i_{ma} = i_{mb} = -\frac{\hat{I}_m}{2}$, $i_{mc} = \hat{I}_m$

$\omega t = 5\pi/3$

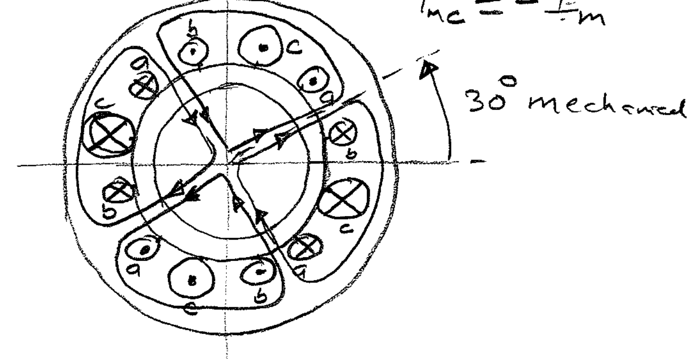
$i_{ma} = i_{mc} = \frac{\hat{I}_m}{2}$; $i_{mb} = -\hat{I}_m$

$\omega t = 0;\ i_{ma} = I_m,\ i_{mb} = i_{mc} = -\frac{I_m}{2}$



$\omega t = 60^\circ;\ i_{ma} = i_{mb} = \frac{I_m}{2}$

$i_{mc} = -I_m$



Comparing the flux-line orientations at the above two instants, we see that in a 60° interval ($1/6$ th of the electrical cycle), the flux-density distribution vector has rotated by an angle of 30° mechanical or $30^\circ \times \frac{p}{2} (= 60^\circ)$ electrical, where $p = 4$.

Therefore, in a 4-pole machine

$$\omega_{syn} \text{ (in rps)} = \frac{2}{p} f = \frac{1}{2} f$$

or,

$$\omega_{syn} \text{ (in rad/s)} = 2\pi \times \frac{1}{2} f$$

$$= \pi f.$$

or,

$$\omega_{syn} \text{ (in rpm)} = 60 \times \frac{1}{2} f$$

$$\equiv 30 f$$

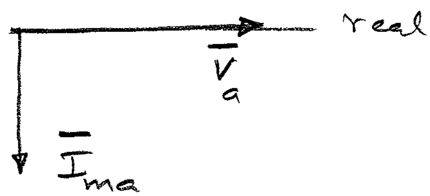
9-12

$$\bar{V}_a = \sqrt{2} \times 120 \angle 0^\circ \ V$$

and,

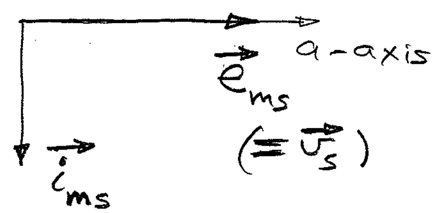
$$\bar{I}_{ma} = \sqrt{2} \times 5 \angle -90^\circ \ A$$

Phasor diagram



Space vector diagram

$\boxed{\text{at } t=0}$



Using Eq 9-34

$$\vec{e}_{ms}\Big|_{t=0} = \frac{3}{2}\bar{E}_{ma}$$

$$= \frac{3}{2}\bar{V}_a \quad \text{(since } R_s \text{ and } L_{\ell s} \text{ are neglected)}$$

$$= \frac{3}{2} \times \sqrt{2} \times 120 \angle 0^\circ \ V$$

Similarly,

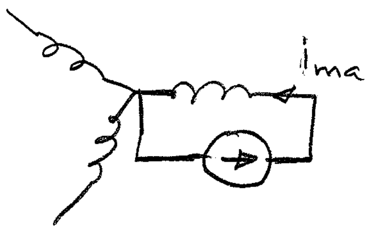
$$\vec{i}_{ms}\Big|_{t=0} = \frac{3}{2}\bar{I}_{ma}$$

$$= \frac{3}{2} \times \sqrt{2} \times 5 \angle -90^\circ$$

Both $\vec{e}_{ms}$ and $\vec{i}_{ms}$ are drawn at $t=0$.

9-13 Prove that $L_{m,\,1\text{-phase}} = \dfrac{\pi \mu_o N_{sp}^2 r l}{l_g}$ in a 2-pole machine, where $N_{sp} = N_s/2$.

With only one phase, for example, phase-a excited by a current i_a (assuming that the neutral is accessible), the flux-density distribution in a 2-pole machine would be



rotor is electrically open-circuited

$$B_{ma}(\theta) = \mu_o \frac{N_{sp}\, i_{ma}}{l_g} \cos\theta$$

∴ Energy density at angle θ would be

$$w_{ma}(\theta) = \frac{1}{2} \frac{B_{ma}^2(\theta)}{\mu_o}$$

∴ The differential energy stored at an angle θ (with respect to a-axis) in a diffential angle $d\theta$ would be

$$dW_{ma}(\theta) = w_{ma}(\theta) \cdot d(\text{volume})$$

$$= \frac{1}{2\mu_o}\left(\frac{\mu_o N_{sp} \cos\theta}{l_g}\right)^2 i_{ma}^2 \; \underbrace{\underbrace{r \cdot d\theta}_{\text{differential arc}} \cdot l \cdot l_g}_{\text{differential volume}}$$

radius, rotor length, airgap length

$$= \frac{\mu_o N_{sp}^2 r l}{2 l_g} i_{ma}^2 \cos^2\theta \cdot d\theta$$

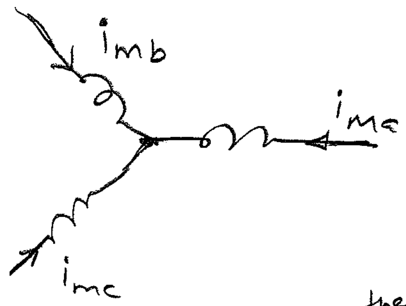
Integrating both side with respect to θ form 0 to 2π:

(total energy stored in the airgap) $$W_{ma} = \frac{\mu_o N_{sp}^2 r l}{2 l_g} i_{ma}^2 \underbrace{\int_0^{2\pi} \cos^2\theta\, d\theta}_{(=\pi)} = \frac{1}{2}\left(\frac{\pi \mu_o N_{sp}^2 r l}{l_g}\right) i_{ma}^2$$

$$= \frac{1}{2} L_{m,\,1\text{-phase}}\, i_{ma}^2$$

$$\therefore L_{m,\,1\text{-phase}} = \frac{\pi \mu_o N_{sp}^2 r l}{l_g} \qquad \underline{\underline{(p=2)}}$$

9-14



at any instant,

$$i_{ma} + i_{mb} + i_{mc} = 0 \quad (1)$$

Leakage fluxes are neglected and the rotor is assumed to be electrically open-circuited.

Flux-linkage λ_{ma}, linking phase-a winding is

$$\lambda_{ma} = L_{m,\,1\text{-phase}}\, i_{ma} + (L_{mutual})_{a-b}\, i_{mb} + (L_{mutual})_{a-c}\, i_{mc} \quad (2)$$

where, by definitions

$$\left.\frac{\lambda_{ma}}{i_{ma}}\right|_{i_{mb}=i_{mc}=0} = L_{m,\,1\text{-phase}}$$

$$\left.\frac{\lambda_{ma}}{i_{mb}}\right|_{i_{ma}=i_{mc}=0} = (L_{mutual})_{a-b}$$

and

$$\left.\frac{\lambda_{ma}}{i_{mc}}\right|_{i_{ma}=i_{mb}=0} = (L_{mutual})_{a-c}$$

Next, we will obtain mutual inductances in terms of $L_{m,\,1\text{-phase}}$. To obtain $(L_{mutual})_{a-b}$, imagine that phase-b winding is on top of phase-a winding, that is, there is no phase shift between the two. In such a case, $(L_{mutual})_{a-b} = L_{m,\,1\text{-phase}}$

But, in reality, phase-b is shifted ahead of phase-a by an angle θ, where $\theta = 120°$. Therefore, the mutual inductance, due to sinusoidally-distributed nature of the two windings varies as the cosine of the angle θ:

$$\left(L_{mutual}\right)_{a-b} = L_{m,1\text{-}phase} \cos\theta,$$

where $\theta = 120°$

$$\therefore \left(L_{mutual}\right)_{a-b} = -\frac{1}{2} L_{m,\,1\text{-}phase} \qquad (3)$$

Similarly,

$$\left(L_{mutual}\right)_{a-c} = L_{m,1\text{-}phase} \cos\theta$$

where $\theta = 240°$

$$\therefore \left(L_{mutual}\right)_{a-c} = -\frac{1}{2} L_{m,1\text{-}phase} \qquad (4)$$

Therefore, substituting Eqs. 3 and 4 into Eq. 2

$$\lambda_{ma} = L_{m,1\text{-}phase}\left(i_{ma} - \frac{1}{2} i_{mb} - \frac{1}{2} i_{mc} \right)$$

$$= \frac{3}{2} L_{m,1\text{-}phase}\, i_{ma} \quad \left[\text{using Eq. 1}\right] \qquad (5)$$

By definition, $$\left.\frac{\lambda_{ma}}{i_{ma}}\right|_{i_{ma}+i_{mb}+i_{mc}=0} = L_m \qquad (6)$$

Therefore, from Eq. 5 into Eq. 6

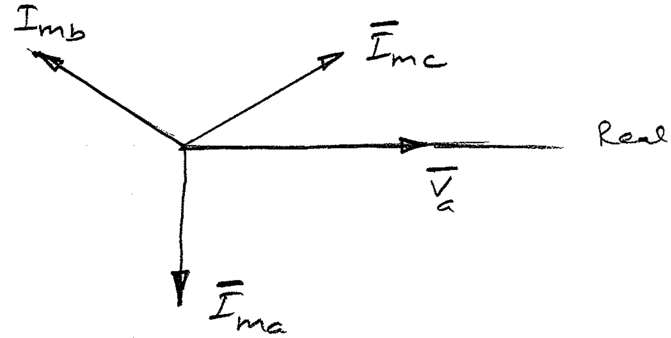
$$L_m = \frac{3}{2} L_{m,1\text{-}phase}\,.$$

9-15

$$\bar{V}_a = \sqrt{2} \times 120 \angle 0^\circ \text{ V}$$

$$X_m = \omega L_m = 2\pi \times 60 \times 75 \times 10^{-3} = 28.28\ \Omega$$

$$\therefore\ \bar{I}_{ma} = \frac{\bar{V}_a}{jX_m} = \sqrt{2} \times 4.24 \angle -90^\circ \text{ A}$$



9-16

Due to $i_{ma}(t)$ at any instant t

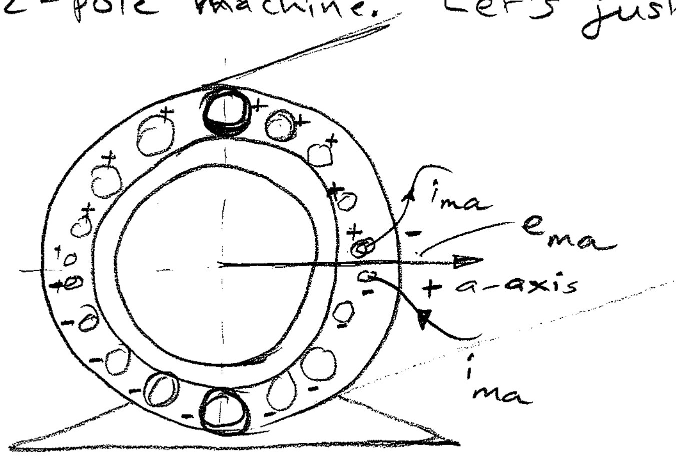
$$\vec{B}_{ma}(t) = \mu_0 \frac{N_s\, i_{ma}(t)}{2\,l_g} \angle 0^\circ$$

and,

$$\vec{B}_{ms}(t) = \frac{3}{2}\, \frac{\mu_0 N_s \hat{I}_m}{2 l_g} \angle \omega t$$, where $\hat{I}_m$ is the peak of per-phase magnetizing currents.

(given that i_{ma} has its positive peak at $t=0$)

2-pole machine. Let's just consider phase-a:



At $\omega t = \pi/2$, $\vec{B}_{ms} = \hat{B}_{ms} \angle 90^\circ$

and the flux lines are vertically up. These flux lines moving at a speed ω_{syn} with respect to the stationary phase-a conductors.

$\therefore$ <u>at $\omega t = \pi/2$</u>

$$B_{ms}(\theta) = \hat{B}_{ms}\cos(\theta - 90^\circ)$$ where θ is

the angle measured with respect to phase-a axis.

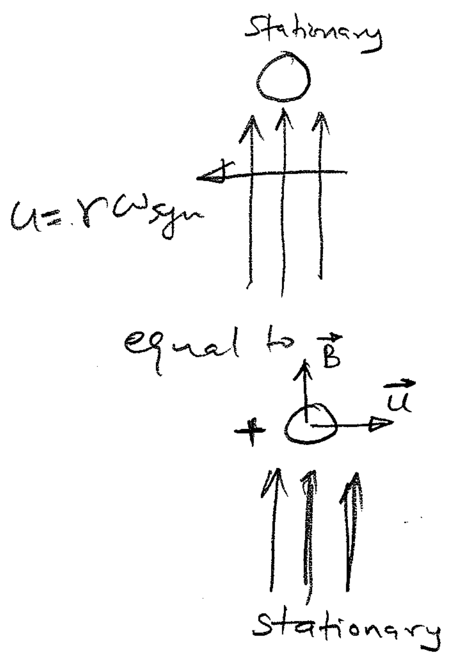
$\therefore$ In a differential angle $d\theta$, at θ, the voltage induced is

$$de_{ma}(\theta) = \underbrace{(N_s/2)\sin\theta \cdot d\theta}_{\substack{\text{differential}\\ \text{number of cond.}}} \; l \cdot \underbrace{r\omega_{syn}}_{u} \; B_{ms}(\theta)$$

$$\therefore \hat{E}_m = \underset{\substack{\uparrow \\ \text{by symmetry}}}{2} \times \left(\frac{N_s}{2}\, l\, r\, \hat{B}_{ms}\,\omega_{syn}\right) \int_0^{\pi} \sin\theta \cdot \underbrace{\cos(\theta - 90^\circ)}_{=\sin\theta}\, d\theta$$

$$= 2\left(\frac{N_s}{2}\right) l\, r\, \hat{B}_{ms}\, \omega_{syn} \underbrace{\int_0^{\pi} \sin^2\theta \cdot d\theta}_{=\pi/2}$$

$$= \left(\frac{N_s}{2}\right) \pi\, l\, r\, \hat{B}_{ms}\, \omega_{syn}$$

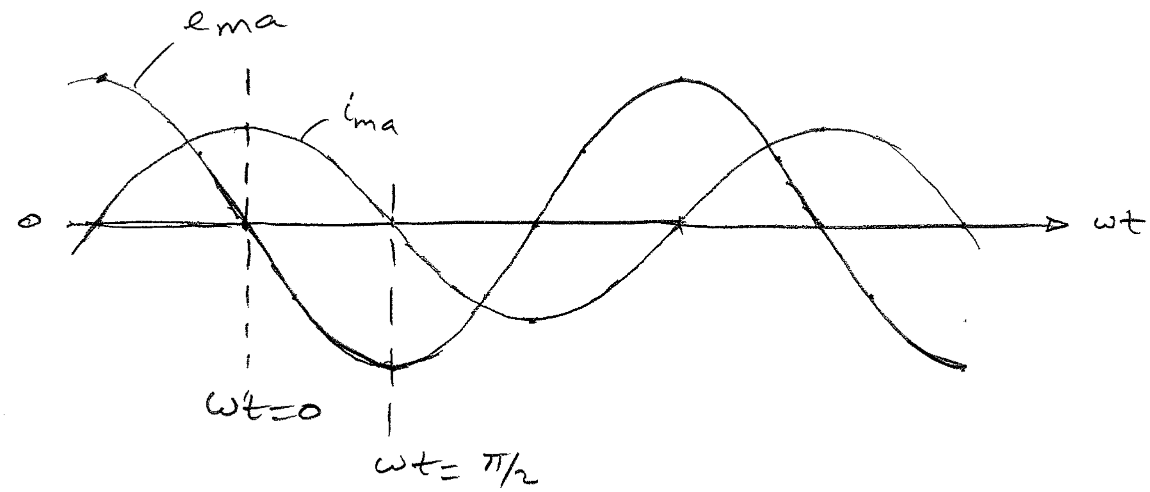


using $\vec{f}_q = q\,\vec{u} \times \vec{B}$ the near-end is positive

Using the analysis shown at the left, the polarity of induced voltages at the near end are shown on the previous page —

$\therefore$ e_{ma}, with the polarity defined on the previous page is at its negative peak — as it should be at $\omega t = \pi/2$!

as shown on the next page



$\therefore$ if $\quad i_{ma}(t) = \hat{I}_m \cos \omega t$

$$e_{ma}(t) = \hat{E}_m \cos\left(\omega t + \frac{\pi}{2}\right)$$

where,
$$\hat{E}_m = \pi l r \frac{N_s}{2} \hat{B}_{ms} \omega_{syn}$$

Substituting for $\hat{B}_{ms}$,

$$\hat{E}_m = \pi l r \frac{N_s}{2} \frac{3}{2} \frac{\mu_o N_{sp}}{l_g} \hat{I}_m \omega_{syn}$$

$$= \left(\frac{3}{2} \pi \mu_o \frac{N_s^2 r l}{4 l_g}\right) \hat{I}_m \omega_{syn}$$

$$= \omega_{syn} L_m \hat{I}_m$$

Substituting the numerical values,

$$\hat{E}_m = 535.8 \text{ V}$$

The other two phase voltages lag phase-a voltage by 120^o and 240^o, respectively.

9-17 In a p-pole machine, assuming that at $\omega t = 0$ $\vec{B}_{ms} = \hat{B}_{ms}\angle 0$

at $\omega t = \pi/2$, $B_{ms}(\theta_e) = \hat{B}_{ms}\cos(\theta_e - \frac{\pi}{2})$

or $B_{ms}(\theta) = \hat{B}_{ms}\cos(\frac{p}{2}\theta - \frac{\pi}{2})$, where θ_e is the electrical angle measured with respect to the phase a-axis,

and $\theta_e = \frac{p}{2}\theta$. Let $\boxed{N_{sp} = \frac{N_s}{p}}$

In a differential angle $d\theta$, at θ, the voltage induced in phase-a is

$$de_{ma}(\theta) = \underbrace{\frac{N_s}{2}\sin(\tfrac{p}{2}\theta)\cdot d\theta}_{\text{differential number of cond.}} \cdot l \cdot \underbrace{r\,\omega_{syn}}_{u} \; B_{ms}(\theta)$$

$$\therefore \hat{E}_m = \underset{\substack{\uparrow \\ \text{by symmetry}}}{p} \times \left(\frac{N_s}{2} l\, r\, \hat{B}_{ms}\,\omega_{syn}\right) \underbrace{\int_0^{2\pi/p} \sin(\tfrac{p}{2}\theta)\cdot \underbrace{\cos(\tfrac{p}{2}\theta - \tfrac{\pi}{2})}_{=\sin(\frac{p}{2}\theta)} \cdot d\theta}_{\left(\pi/2 \,/\, \frac{p}{2}\right)}$$

$$= p \times \frac{N_s}{2}\, l\, r\, \hat{B}_{ms}\,\omega_{syn}\,\frac{\pi}{2}\left(\frac{2}{p}\right)$$

$\omega_{syn} = \frac{2}{p}\omega$, where $\omega = 2\pi f$. Substituting for ω_{syn},

$$\hat{E}_m = \cancel{p} \times \frac{N_s}{\cancel{2}} \times l\, r\, \hat{B}_{ms} \times \left(\frac{\cancel{2}}{\cancel{p}}\omega\right) \times \frac{\pi}{\cancel{2}}\,\frac{2}{p}$$

$$= (\pi\, r\, l\, N_{sp})\,\omega\,\hat{B}_{ms}, \qquad N_{sp} = \frac{N_s}{p}$$

$$\therefore \hat{E}_{ms} = \omega\left(\tfrac{3}{2}\pi\, r\, l\, N_{sp}\right)\hat{B}_{ms}$$

and $$\vec{E}_{ms}(t) = j\omega\left(\tfrac{3}{2}\pi\, r\, l\, N_{sp}\right)\vec{B}_{ms}(t)$$

9-18

Calculate L_m in a p-pole machine ($p \geq 2$)

<u>Solution</u> Let $N_{sp} = \dfrac{N_s}{p}$

from Eq. 9-12 b

$$B_{ma}(\theta) = \mu_o \frac{N_{sp}}{\ell_g} i_{ma} \cos\left(\frac{p}{2}\theta\right) \qquad (1)$$

$\therefore$ Energy density at θ is

$$w_{ma}(\theta) = \frac{1}{2\mu_o} B_{ma}^2(\theta)$$

and the differential energy at angle θ in a differential angle $d\theta$ will be

$$dw_{ma}(\theta) = w_{ma}(\theta)\, d(\text{volume})$$

$$= \frac{1}{2\mu_o} B_{ma}^2(\theta) \underbrace{(r \cdot d\theta)}_{\text{note: } \theta,\ \underline{\text{not}}\ \theta_e} \ell \cdot \ell_g$$

$$\therefore W_{ma} = \underset{\substack{\uparrow \\ \text{by symmetry}}}{p} \times \int_0^{2\pi/p} \frac{1}{2\mu_o} \mu_o^2 \frac{N_{sp}^2}{\ell_g^2} i_{ma}^2 \cos^2\left(\frac{p}{2}\theta\right) r\, \ell\, \ell_g\, d\theta$$

$$= \frac{p}{2} \mu_o \frac{N_{sp}^2}{\ell_g} r \ell \cdot i_{ma}^2 \int_0^{2\pi/p} \cos^2\left(\frac{p}{2}\theta\right) \cdot d\theta$$

$\frac{p}{2}\theta = \theta_e$

and $d\theta = \frac{2}{p}\, d\theta_e$

$$\therefore W_{ma} = \frac{p}{2} \mu_o \frac{N_{sp}^2}{\ell_g} r \ell\, i_{ma}^2 \frac{2}{p} \underbrace{\int_0^{\pi} \cos^2\theta_e \cdot d\theta_e}_{\pi/2}$$

$$= \frac{\cancel{p}}{2} \mu_o \frac{N_{sp}^2}{\ell_g} r \ell\, i_{ma}^2 \frac{\cancel{2}}{\cancel{p}} \frac{\pi}{\cancel{2}}$$

$$= \frac{1}{2} \pi r \ell \mu_o \frac{N_{sp}^2}{\ell_g} i_{ma}^2 = \frac{1}{2} L_{m,1\text{-phase}}\, i_{ma}^2$$

$$\therefore \quad L_{m,\,1\text{-phase}} = \frac{\pi r \ell}{\ell_g} \mu_o N_{sp}^2$$

and

$$\therefore \quad L_m = \frac{3}{2} \frac{\pi r \ell}{\ell_g} \mu_o N_{sp}^2 \qquad p \geq 2$$

where $\boxed{N_{sp} = \frac{N_s}{p}}$

9-19

Combine the results of Problems 9-17 and 9-18 to show that (for $p \geq 2$), $\vec{e}_{ms}(t) = j\omega L_m \vec{i}_{ms}(t)$.

Solution

From the solution of Problem 9-17,

$$\vec{e}_{ms}(t) = j\omega \left(\frac{3}{2} \pi r \ell N_{sp}\right) \vec{B}_{ms}(t)$$

Substituting for $\vec{B}_{ms}(t)$ from Eq. 9-12 b,

$$\vec{e}_{ms}(t) = j\omega \left(\frac{3}{2} \pi r \ell N_{sp}\right) \left(\frac{\mu_o N_{sp}}{\ell_g}\right) \vec{i}_{ms}(t)$$

$$= j\omega \underbrace{\left(\frac{3}{2} \frac{\pi r \ell}{\ell_g} \mu_o N_{sp}^2\right)}_{= L_m \text{ from Problem 9-18}} \vec{i}_{ms}(t)$$

$$\therefore \quad \vec{e}_{ms}(t) = j\omega L_m \vec{i}_{ms}(t) \qquad p \geq 2$$

9-20

In parts (b) through (g) the flux-density distribution peaks at the angle θ as shown in Fig. 9-15. It varies cosinusoidally from that peak, as a function of θ.

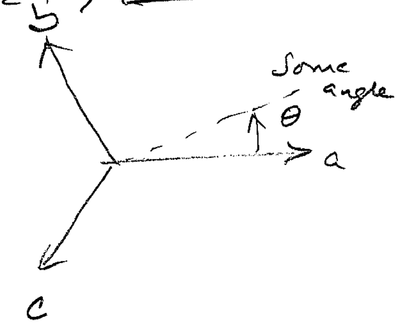
9-21

at sometime t, $\vec{B}_s = 1.1\angle 30^\circ$. Using Eqs. 9.24, at this time

$$\vec{B}_{s,a} = \frac{2}{3}(1.1)\cos 30^\circ \angle 0^\circ$$

$$\vec{B}_{s,b} = \frac{2}{3}(1.1)\cos(30^\circ - 120^\circ)\angle 120^\circ$$

$$\vec{B}_{s,c} = \frac{2}{3}(1.1)\cos(30^\circ - 240^\circ)\angle 240^\circ$$



$$\therefore B_{s,a}(\theta) = \hat{B}_{s,a}\cos(\theta - 0^\circ)$$

$$B_{s,b}(\theta) = \hat{B}_{s,b}\cos(\theta - 120^\circ)$$

$$B_{s,c}(\theta) = \hat{B}_{s,c}\cos(\theta - 240^\circ)$$

where,

$$\hat{B}_{s,a} = \frac{2}{3}(1.1)\cos 30^\circ = 0.635\ T$$

$$\hat{B}_{s,b} = \frac{2}{3}(1.1)\cos 90^\circ = 0\ T$$

$$\hat{B}_{s,c} = \frac{2}{3}(1.1)\cos(-210^\circ) = -0.635$$

9-22

In Fig. 9.29, assume that B_{ms} distribution peak at $\theta = -90^\circ$ at time $t=0$. That is,

$\vec{B}_{ms} = \hat{B}_m e^{-j90^\circ}$. This flux-density distribution is rotating CCW at a speed ω rad/s.

Therefore, in phase-a, the peak voltage occurs at this time. At an angle ξ

$$\hat{E}_{ph} = 2 \times \int_0^{\pi} \hat{B}_{ms} \sin\xi \, \frac{N_s}{2} \sin\xi \, l r \omega \cdot d\xi$$

$$= 2 \times \left(\hat{B}_{ms} \frac{N_s}{2} l r \omega \right) \int_0^{\pi} \sin^2\xi \cdot d\xi$$

$$= 2 \times \hat{B}_{ms} \frac{N_s}{2} l r \omega \times \pi/2$$

$$\vec{e}_{ms} = \hat{E}_{ph} \angle 0^\circ + \hat{E}_{ph} \cos 120 \angle +120^\circ + \hat{E}_{ph} \cos 240 \angle +240^\circ$$

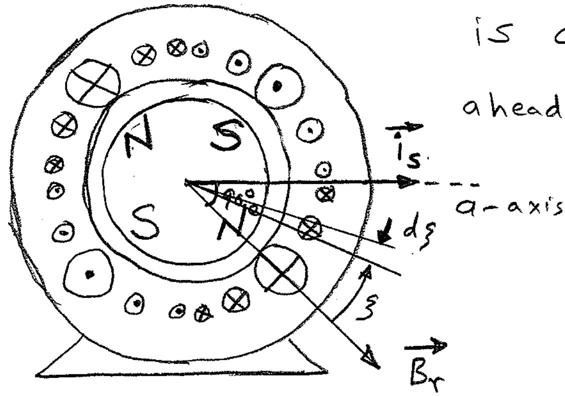
$$= \frac{3}{2} \hat{E}_{ph} \angle 0^\circ$$

$$\vec{e}_{ms}(t) = j\omega \left(\frac{3}{2} \pi r l \frac{N_s}{2} \right) \vec{B}_{ms}(t)$$

Chapter 10

10-1

A 4-pole machine is shown below at the instant when $\vec{i}_s$ vector is along the a-axis, 90° (electrical) ahead of the $\vec{B}_r$ space vector in a CCW direction.



drawn for p=4

Let $N_{sp} = \frac{N_s}{p} = \frac{N_s}{4}$ (since $p=4$)

Following Eqs. 10-3 through 10-6 in the text, in actual (mechanical) angles with respect to the position of the $\vec{B}_r$ space vector, for a p-pole machine

$$dT_{em}(\xi) = r \cdot \underbrace{\hat{B}_r \cos(\tfrac{p}{2}\xi)}_{\text{flux density at } \xi} \cdot l \cdot \hat{I}_s \cdot \underbrace{\frac{N_s}{2}\cos(\tfrac{p}{2}\xi)\cdot d\xi}_{\text{diff. no. of conductors at } \xi}$$

Integrating from $\xi = -\pi/p$ to π/p, and then by symmetry multiplying by a factor of p $\boxed{N_{sp} = \frac{N_s}{p}}$

$$T_{em} = p \times r l \frac{N_s}{2} \hat{B}_r \hat{I}_s \underbrace{\int_{-\pi/p}^{\pi/p} \cos^2(\tfrac{p}{2}\xi)\cdot d\xi}_{=\frac{2}{p}(\frac{\pi}{2})} = \left(\frac{p}{2}\pi r l N_{sp} \hat{B}_r\right)\hat{I}_s = k_T \hat{I}_s$$

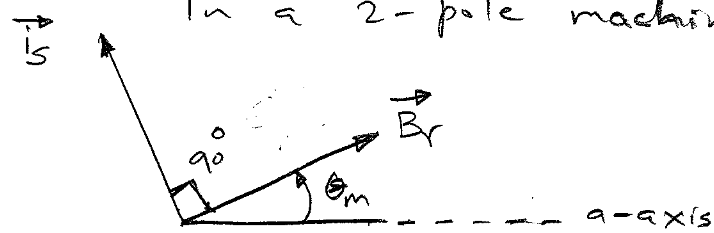
$$\therefore \; k_T = \frac{p}{2} \times \pi r l N_{sp} \hat{B}_r = \frac{\cancel{p}}{2} \times \pi r l \frac{N_s}{\cancel{p}} \hat{B}_r$$

$$= \pi r l \frac{N_s}{2} \hat{B}_r$$

Note that k_T is independent of the number of poles, provided we express it in terms of N_s (the total number of turns per-phase).

10-2

In a 2-pole machine, as shown below

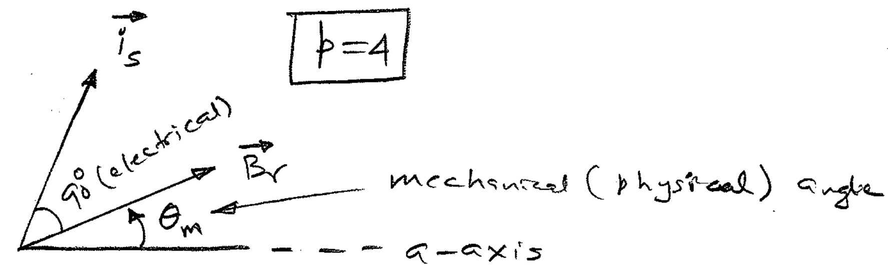


where

$$\theta_{i_s}^* = \theta_m + \frac{\pi}{2} \quad (\text{for a CCW torque}),$$ where

are the angles are mechanical (physical).

In a p-pole machine ($p=4$, for example) shown below



$\therefore$ In electrical angles, with respect to a-axis

$$\theta_{i_s}^*(t) = \frac{p}{2}\theta_m + \frac{\pi}{2} \quad (\text{for a CCW torque}),$$

Same as that given by Eq. 10-11, where

$$i_a^*(t) = \frac{2}{3}\hat{I}_s^* \cos\theta_{i_s}^*(t)$$

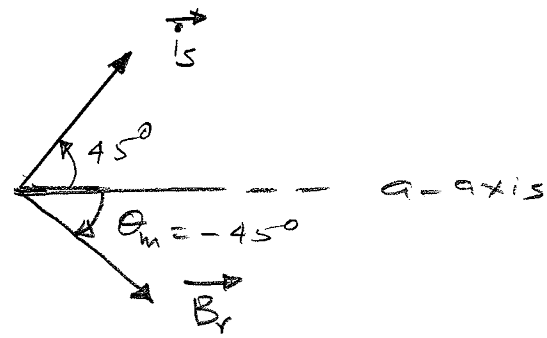
Note that for a variation of θ_m by 2π radians, $\theta_{i_s}^*$ varies by $\frac{p}{2}(2\pi)$ radians, that is, $i_a^*(t)$ goes through $\frac{p}{2}$ cycles.

10-3

From Eq. 10-10

$$\theta_{i_s} = \theta_m + 90^\circ = -45^\circ + 90^\circ = 45^\circ$$

Therefore, $\vec{i}_s(t) = \hat{I}_s \angle \theta_{i_s} = 10\angle 45^\circ$



$$\therefore \quad i_a = \frac{2}{3}\hat{I}_s \cos\theta_{i_s} = 4.71\text{ A}$$

$$i_b = \frac{2}{3}\hat{I}_s \cos(\theta_{i_s} - 120^\circ) = 1.73\text{ A}$$

$$i_c = \frac{2}{3}\hat{I}_s \cos(\theta_{i_s} - 240^\circ) = -6.44\text{ A}$$

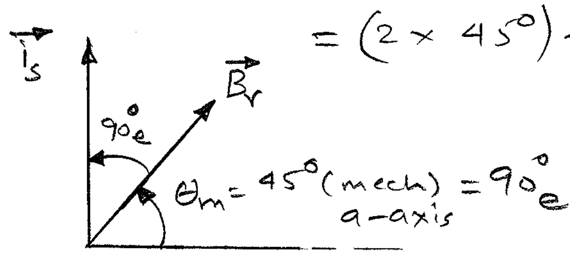
10-4

$p = 4$ poles, $\quad k_T = 0.5 \quad \therefore \hat{I}_s = 10\text{ A}.$

From Eq. 10-11, for $\theta_m = 45^\circ$

$$\theta_{i_s} = \frac{p}{2}\theta_m + 90^\circ$$

$$= (2 \times 45^\circ) + 90^\circ = 180^\circ$$

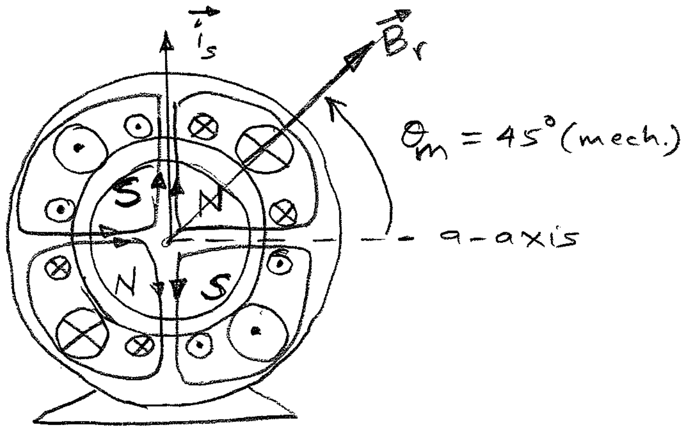


$$\therefore \quad i_a = \frac{2}{3}\hat{I}_s \cos\theta_{i_s} = -6.66\text{ A}$$

$$i_b = \frac{2}{3}\hat{I}_s \cos(\theta_{i_s} - 120^\circ) = 3.33\text{ A}$$

$$i_c = \frac{2}{3}\hat{I}_s \cos(\theta_{i_s} - 240^\circ) = 3.33\text{ A}$$

This corresponds to the distribution shown on the next page.



10-5

$$T_L = 5\ Nm \quad \therefore \quad \hat{I}_s = \frac{5}{k_T} = 10\,A \quad \text{and} \quad \hat{I}_a = \frac{2}{3}\hat{I}_s = 6.67A$$

$$\text{speed} \quad \omega_m = \frac{5000}{60} \times 2\pi = 523.6\ \text{rad/s}$$

From Eq. 10-22

$$\hat{E}_f = k_E\,\omega_m = 0.5 \times 523.6 = 261.8\ V$$

Assuming $\theta_m(0) = -90^\circ$, from Eq. 10-10, $\theta_{i_s}\Big|_{t=0} = 0^\circ$

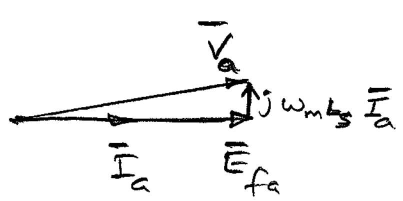
$$\therefore \quad i_a = \hat{I}_a \cos\omega_m t \Rightarrow \bar{I}_a = 6.67\angle 0^\circ$$

$$\text{and} \quad \bar{E}_{fa} = 261.8\angle 0^\circ$$

$$\bar{V}_a = \bar{E}_{fa} + j\omega_m L_s \bar{I}_a$$

$$= 261.8 + j\,523.6 \times (15 \times 10^{-3}) \times 6.67\angle 0^\circ$$

$$= 267.0\angle 11.29^\circ\ V$$



10-6 $\qquad p = 4$, $\qquad \omega_m = 523.6 \ \frac{\text{rad (mech.)}}{s}$ ← rotor mech. speed

$$\omega_e = \frac{p}{2} \times 523.6 \ \frac{\text{rad}}{s} \ \text{(electrical)}$$

$$= 1047.2 \ \text{rad (elect.)}/s$$

↑ speed at which the distribution is rotating

$$k_T = \frac{T_{em}}{\hat{I}_s} = \underbrace{\frac{\pi}{2} r \ell N_s \hat{B}_r}_{\text{from solution to Problem 10-1}} = 0.5 \ \frac{Nm}{A}$$

↑ given

From the solution to Problem 9-17 of the previous chapter, in a p-pole machine

$$\hat{E}_m = \pi r \ell \frac{N_s}{p} \omega \hat{B}_{ms}$$

$$\omega_{syn} = \frac{2}{p} \omega$$

$$\therefore \omega = \frac{p}{2} \omega_{syn}$$

$$= \frac{\pi}{2} r \ell N_s \omega_{syn} \hat{B}_{ms}$$

Adapting it to the PMAC Drive

$$\hat{E}_m \Rightarrow \hat{E}_f$$

$$\hat{B}_{ms} \Rightarrow \hat{B}_r$$

$$\omega_{syn} \Rightarrow \omega_m$$

$$\therefore \quad \hat{E}_f = \underbrace{\frac{\pi}{2} r \ell N_s \hat{B}_r}_{k_E \ (= k_T)} \omega_m$$

↑ mech. rad/s

$$\therefore \quad k_E = \frac{\hat{E}_f}{\omega_m} = 0.5 \ \frac{V}{\text{mech. rad/s}}$$

$\therefore$

$$\hat{E}_f = k_E \omega_m = 0.5 \times 523.6 = 261.8 \text{ V}$$

Assuming $\theta_m(0) = \frac{90^\circ}{p/2} = 45^\circ$, from Eq. 10-11, $\theta_{i_s}\big|_{t=0} = 0^\circ$,

and from Eq. 10-13

$$\theta_{i_s}(t) = \frac{p}{2}\omega_m t = \omega_e t = 1047.2 \times t \quad \text{rad}$$

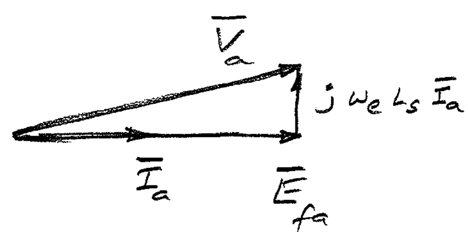
$$\therefore \quad i_a(t) = \hat{I}_a \cos(\theta_{i_s}(t)) = 6.67 \cos(\theta_{i_s}(t))$$

$$\Rightarrow \quad \bar{I}_a = 6.67\angle 0^\circ \text{ A}$$

$$\bar{V}_a = \bar{E}_{fa} + j\omega_e L_s \bar{I}_a$$

$$= 261.8 + j\,1047.2 \times (15 \times 10^{-3}) \times 6.67\angle 0^\circ$$

$$\simeq 282.0\angle 21.83^\circ \text{ V}$$



10-7

In Problem 10-5, $\theta_m(0) = 90^\circ$

if $\theta_m(0) = 0^\circ \Rightarrow \theta_{i_s}\big|_{t=0} = 90^\circ$

$\therefore \quad i_a = \hat{I}_a \cos(\omega_m t + 90^\circ) \Rightarrow \bar{I}_a = 6.67\angle +90^\circ$ A

$\bar{E}_{fa} = 261.8\angle +90^\circ$ V; Correspondingly $\bar{V}_a = 267.0\angle(11.29 + 90)^\circ$

$= 267.0\angle 101.29^\circ$ V

10-8

$$T_{em} = 5 \text{ Nm}$$

$$\omega_m = 0 \text{ to } 5000 \text{ rpm in } 5s$$

Since the torque developed is constant at 5 Nm, the acceleration $\left(\frac{d\omega_m}{dt}\right)$ is also a constant. Therefore,

$$\omega_m(t) = \frac{\Delta\omega_m}{\Delta t} \cdot t = \frac{\frac{5000}{60} \times 2\pi - 0}{5} t$$

$$= \frac{523.6}{5} t = 104.72 t \quad \frac{\text{rad}}{s}$$

Assuming that $\theta_m(0) = -90°$, $\theta_{i_s}\big|_{t=0} = 0°$

$$\therefore \theta_{i_s}(t) = \int_0^t \omega_m(\tau) d\tau = 104.72 \int_0^t \tau = 52.36 t^2$$

$$\hat{I}_s = \frac{T_{em}}{k_T} = \frac{5.0}{0.5} = 10 \text{ A} \quad \therefore \hat{I}_a = \frac{2}{3} \hat{I}_s = 6.67 \text{ A}$$

$0 \leq t \leq 5$

$$\therefore i_a(t) = \hat{I}_a \cos(\theta_{i_s}(t)) = 6.67 \cos(52.36 t^2) \quad 0 < t < 5s$$

$$\hat{E}_f = k_E \omega_m(t) = 0.5 \times 104.72 t = 52.3 t$$

$$\therefore e_{fa}(t) = \hat{E}_f \cos(\theta_{i_s}(t)) = 52.3 t \cos(52.36 t^2)$$

Armature reaction:

$$L_s \frac{di_a}{dt} = L_s \times \frac{d}{dt}\left[6.67 \cos(52.36 t^2)\right]$$

$$= 15 \times 10^{-3} \times 6.67 \left[-2t \sin(52.36 t^2)\right]$$

$$= -0.2 t \sin(52.36 t^2)$$

$$\therefore v_a(t) = e_{fa}(t) + L_s \frac{di_a}{dt} = 52.3 t \cos(52.36 t^2) - 0.2 t \sin(52.36 t^2) \text{ V}$$

10-9

$$\theta_m\Big|_{t=5s} = 0$$

To produce to a regenerative (clockwise) torque,

$$\theta_{i_s}\Big|_{t=5^+s} = \theta_m\Big|_{t=5s} - 90^\circ = -90^\circ$$

$\hat{I}_a = 6.67\ A$ as in Problem 10-8

$$\therefore \quad i_a(t=5^+s) = \hat{I}_a \cos\left(\theta_{i_s}\Big|_{t=5^+s}\right) = 0\ A$$

$$i_b(t=5^+s) = \hat{I}_a \cos\left(\theta_{i_s}\Big|_{t=5^+s} - 120^\circ\right)$$

$$= 6.67 \cos(-90^\circ - 120^\circ) = -5.78\ A$$

and,

$$i_c(t=5^+s) = \hat{I}_a \cos\left(\theta_{i_s}\Big|_{t=5^+s} - 240^\circ\right)$$

$$= 6.67 \cos(-90^\circ - 240^\circ) = 5.78\ A$$

Problem 10-10

In the generator mode, $\vec{i_s}$ space vector would be lagging behind $\vec{B_r}$ by 90^0 in the direction of rotation of these vectors.

Problem 10-11

$$\bar{V}_a = \bar{E}_{fa} - j\omega_m L_s \bar{I}_a$$

$$= 157.08 - j\,18.85$$

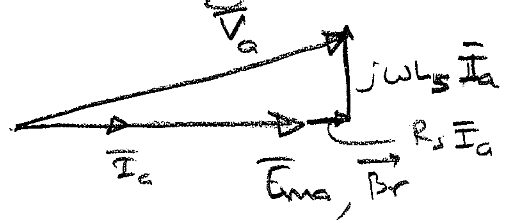
$$= 158.2 \angle -6.84^\circ \text{ V}$$

Problem 10-12

$R_S = 0.416\ \Omega$, $L_s = 1.365$ mH, $k_T = 0.0957\ \frac{Nm}{A}$

$T_{em} = 3.2$ Nm and speed $= 6000$ rpm

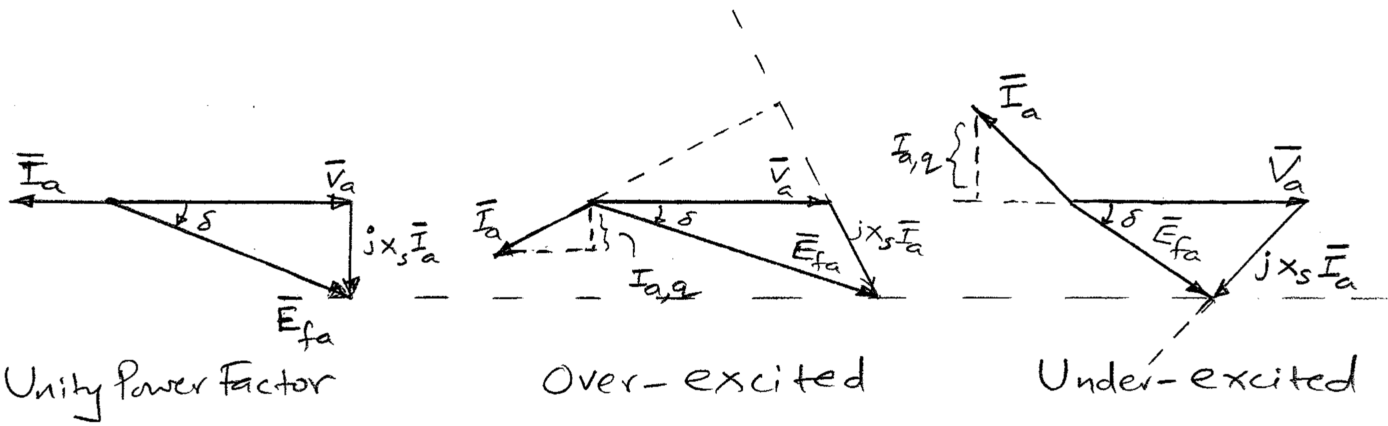
It is very similar to Example 10.2 for finding the phasors:



Then, the space vectors $\vec{v_s}$ and $\vec{i_s}$ have the same orientation.

10-13

In the motoring mode, $\delta = -$. The following phasor diagrams are drawn for a constant power, therefore, $\hat{E}_f \sin\delta$ is constant.



Since the power is constant, the projection of $\bar{I}_a$ on the real axis (along $\bar{V}_a$ phasor) should be the same.

In the over-excited case, $I_{a,q}$ component of $\bar{I}_a$ lags $\bar{V}_a$. Therefore, the utility is absorbing reactive power as an inductor does, and the synchronous motor is supplying reactive power like a capacitor does.

In the under-excited case, $I_{a,q}$ component of $\bar{I}_a$ leads $\bar{V}_a$. Therefore, the motor is absorbing reactive power as an inductor does.

10-14

In a synchronous generator, the complex power from Eq. 10-25,

$$S = \frac{3}{2} V_a I_a^* = \frac{3}{2} \hat{V} \left[\frac{\hat{E}_f \sin\delta}{X_s} + j \frac{\hat{E}_f \cos\delta - \hat{V}}{X_s} \right]$$

Since $S = P + jQ$

$$Q = \frac{3}{2} \hat{V} \left(\frac{\hat{E}_f \cos\delta - \hat{V}}{X_s} \right)$$

$$= \frac{3}{2} \frac{\hat{V}}{X_s} \left(\hat{E}_f \cos\delta - \hat{V} \right)$$

At $\hat{E}_f \cos\delta = \hat{V}$, $Q = 0$, unity power factor operation

if $\hat{E}_f \cos\delta > \hat{V}$, $Q = +$, over-excited case and the generator supplies reactive power, like a capacitor does.

if $\hat{E}_f \cos\delta < \hat{V}$, $Q = -$, under-excited case and the synchronous generator absorbs reactive power, like an inductor does.

Chapter 11

11-1

$$\hat{V}_a = \frac{208}{\sqrt{3}}\sqrt{2} \ V$$

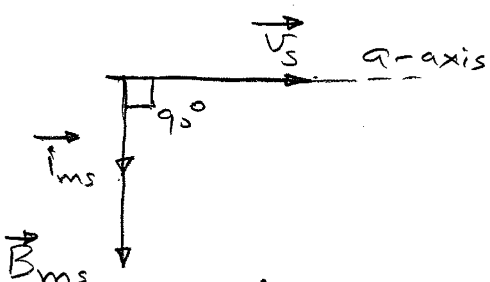
$$\therefore \ \hat{V}_s = \frac{3}{2}\hat{V}_a = \frac{3}{2}\frac{208\sqrt{2}}{\sqrt{3}} = 254.75 \ V$$

at $\omega t = 0$, v_a is at its positive peak

$$\therefore \ \vec{v}_s = \hat{V}_s \angle 0 = 254.75\angle 0^\circ \ V$$

$\boxed{\omega t = 0}$

$$\vec{i}_{ms} = \frac{\vec{v}_s}{j\omega L_m} = \frac{254.75\angle 0^\circ}{j\,377 \times 60 \times 10^{-3}} = 11.26\angle -90^\circ \ A$$



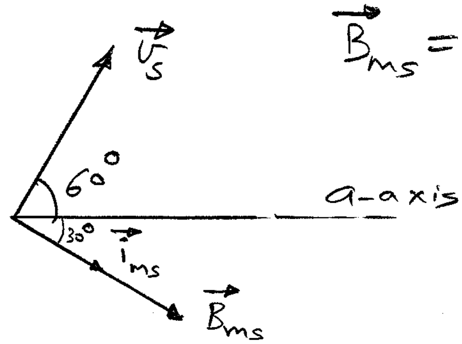
$$\vec{B}_{ms} = 0.85\angle -90^\circ \ T$$

At $\omega t = \pi/3$ (60°), all space vectors would have rotated by 60°, compared to at $\omega t = 0^\circ$:

$$\vec{v}_s = 254.75\angle 60^\circ \ V$$

$$\vec{i}_{ms} = 11.26\angle -30^\circ \ A$$

$$\vec{B}_{ms} = 0.85\angle -30^\circ \ T$$



$$\hat{I}_m = \frac{2}{3}\hat{I}_{ms}$$

and,

$$\hat{B}_{ms} = \mu_0 \frac{N_{sp}}{\ell_g}\hat{I}_{ms}$$

11-2

$$\omega_{syn} = \frac{2}{p}\,\omega \qquad \text{where} \quad \omega = 2\pi f = 377 \text{ rad/s}$$

$\therefore$ $p = 2 \Rightarrow \omega_{syn} = 377$ rad/s $\Rightarrow$ 3,600 rpm

$p = 4 \Rightarrow \omega_{syn} = 188.5$ rad/s $\Rightarrow$ 1,800 rpm

$p = 6 \Rightarrow \omega_{syn} = 125.67$ rad/s $\Rightarrow$ 1,200 rpm

$p = 8 \Rightarrow \omega_{syn} = 94.25$ rad/s $\Rightarrow$ 900 rpm

$p = 12 \Rightarrow \omega_{syn} = 62.83$ rad/s $\Rightarrow$ 600 rpm

11-3 At the rated torque, slip $s = 0.04$. $f = 60$ Hz

Calculate ω_{slip} and f_{slip}

(a) $f_{slip} = sf = 0.04 \times 60 = 2.4$ Hz, regardless of the number of poles.

(b) $\omega_{slip} = \frac{2}{p} \times 2\pi \times f_{slip} = \frac{4\pi}{p} f_{slip}$ (mech. rad/s)

p	ω_{syn}	ω_{slip}	ω_m
2	377.0	15.08	361.92
4	188.5	7.54	180.96
6	125.67	5.03	120.64
8	94.25	3.77	90.48
12	62.83	2.51	60.32

← all in mech. rad/s

In each case, $\omega_{slip} = s \times \omega_{syn}$ (in mech. rad/s)

11-4

By Faraday's Law

$$v_1 = N_1 \frac{d\phi_m}{dt}$$

$$\therefore \phi_m(t) = \frac{1}{N_1}\int v_1(t)\cdot dt$$

$$= \underbrace{\frac{\hat{V}_1}{\omega N_1}}_{\hat{\phi}_m} \sin\omega t$$

$$\therefore \hat{\phi}_m = \frac{\hat{V}_1}{\omega N_1} = \frac{\sqrt{2}\times\cancel{100}}{377\times\cancel{100}} = \frac{\sqrt{2}}{377} = 3.75\times10^{-3}\ \text{Wb.}$$

$$e_2 = N_2\frac{d\phi_m}{dt}$$

$$= \frac{v_1}{2} = 50\sqrt{2}\cos\omega t$$

From Ampere's Law

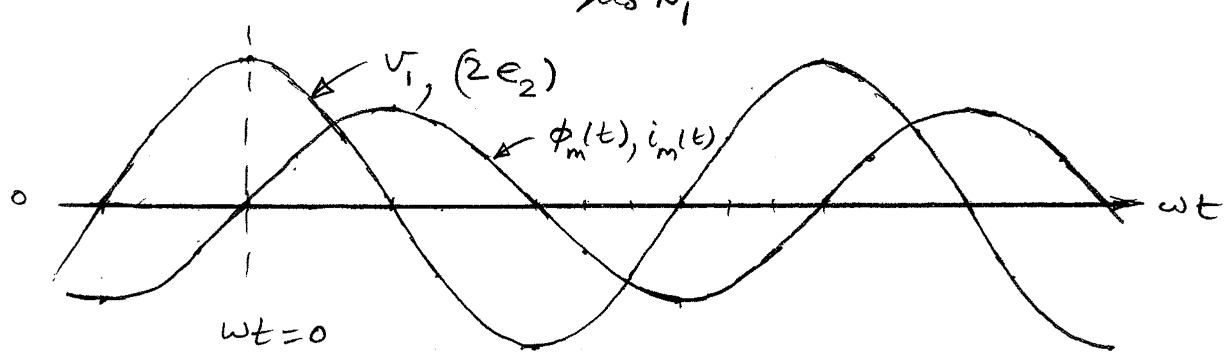
$$2H_g l_g = N_1 i_m$$

$$\therefore H_g = \frac{N_1 i_m}{2l_g}$$

$$B_g = \mu_0 H_g = \mu_0\frac{N_1 i_m}{2l_g}$$

and $$\hat{B}_g = \frac{\mu_0 N_1 \hat{I}_m}{2l_g} = 1.1\ \text{T}$$

$$\therefore \hat{I}_m = \frac{2\times\hat{B}_g\times l_g}{\mu_0 N_1} = 17.51\ \text{A}$$



11-5 Example 11-1

From Eq. 9-36

$$L_m = \frac{3}{2}\left(\frac{\pi \mu_o N_s^2 r \ell}{4 \ell_g}\right)$$

$$\therefore L_m = \frac{3}{2} \frac{\pi \mu_o \times 57^2 \times 0.07 \times 0.09}{4 \times 0.5 \times 10^{-3}}$$

$$= 60.6 \text{ mH}$$

11-6

Eq. 11-30 $T_{em} = k_{TW} \omega_{slip}$ $\underline{B_{ms} = \text{rated}}$

Eq. 11-41 $T_{em} = 3R_r' \dfrac{(I_{ra}')^2}{\omega_{slip}}$

Substituting for T_{em} from Eq. 11-30 into Eq. 11-41,

$$k_{TW} \omega_{slip} = 3R_r' \frac{(I_{ra}')^2}{\omega_{slip}}$$

$$\therefore (I_{ra}')^2 = \frac{k_{TW}}{3R_r'} \omega_{slip}^2$$

or,

$$I_{ra}' = \sqrt{\frac{k_{TW}}{3R_r'}}\,\omega_{slip}$$ [Note: I_{ra}' is the rms value]

$$\therefore \hat{I}_r' = \frac{3}{2}\sqrt{2}\, I_{ra}' = \frac{3}{\sqrt{2}} I_{ra}' = \frac{3}{\sqrt{2}} \frac{1}{\sqrt{3}} \sqrt{\frac{k_{TW}}{R_r'}} = \sqrt{\frac{3}{2} \frac{k_{TW}}{R_r'}}$$

(to convert from rms to peak)

11-7

Eq. 11-41

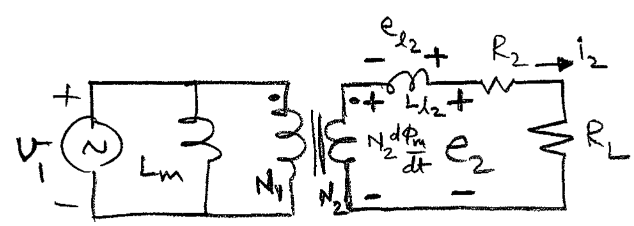
$$T_{em} = 3 R_r' \frac{(I_{ra}')^2}{\omega_{slip}}$$

At the rated torque, I_{ra}' is at its rated value.

Therefore, $\frac{R_r'}{\omega_{slip}}$ should have the same value.

In case where R_r' is doubled, ω_{slip} would also be of twice the value of the original case.

11-8



In the circuit above, it is clear that e_2 is in phase with i_2 because the two are related by the Ohm's Law:

$$e_2(t) = (R_2 + R_L)\, i_2(t)$$

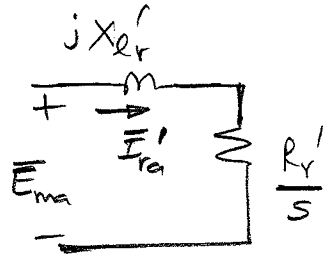
However, $e_2 = N_2 \frac{d\phi_m}{dt} + \underbrace{N_2 \frac{d\phi_{\ell 2}}{dt}}_{\left(e_{\ell 2} = L_{\ell 2} \frac{di_2}{dt}\right)}$

ϕ_m is created by the applied voltage V_1.

$\phi_{\ell 2}$ is the secondary leakage flux created by flowing of i_2.

Where, e_2 is voltage induced in the secondary winding by the time derivative of the combination of ϕ_m and $\phi_{\ell 2}$.

11-9



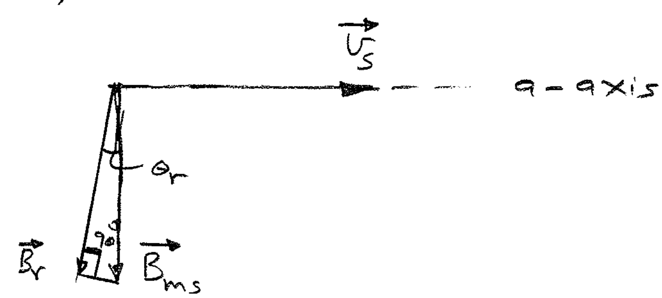
At the rated torque, $s = 0.04$

$\therefore$ In the rotor circuit

$$\tan \theta_r = \frac{X_{\ell r}'}{R_r'/s} = \frac{0.83}{(0.45/0.04)} = 0.074$$

$$\therefore \theta_r = 4.2^\circ$$

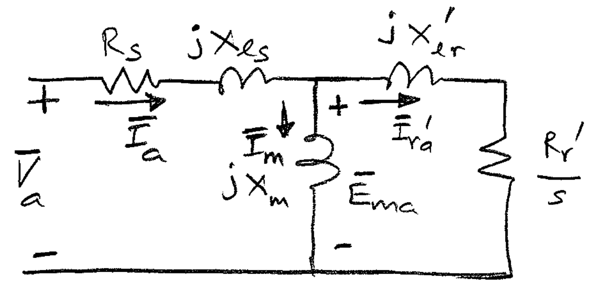
In Fig. 11-17 c,



$$\hat{B}_r = \hat{B}_{ms} \cos \theta_r$$

$$\therefore \frac{\hat{B}_r}{\hat{B}_{ms}} = \cos \theta_r = 0.997$$

11-10



We will use rms values to represent all phasors.

$$\therefore \quad \bar{V}_a = \frac{208}{\sqrt{3}}\angle 0^\circ = 120.0\angle 0^\circ \text{ V}$$

At the terminals, the impedance is

$$Z = (R_s + jX_{ls}) + \frac{(jX_m)(jX'_{lr} + R'_r/s)}{\frac{R'_r}{s} + j(X_m + X'_{lr})}$$

$$= (0.5 + j0.6) + \frac{(j28.5)(j0.83 + 11.25)}{11.25 + j(29.33)}$$

$$= 10.95\angle 26.9^\circ \ \Omega$$

$$\therefore \quad \bar{I}_a = \frac{\bar{V}_a}{Z} = \frac{120\angle 0^\circ}{10.95\angle 26.9^\circ} = 10.96\angle -26.9^\circ \text{ A}$$

Input Power factor $= \cos(26.9^\circ) = 0.89$ (lagging)

$$\bar{I}'_{ra} = \bar{I}_a \frac{jX_m}{\frac{R'_r}{s} + j(X_m + X'_{lr})} = 9.94\angle -5.9^\circ \text{ A}$$

$$\therefore \quad P_{r,loss\ 3\text{-}\phi} = 3R'_r(I'_{ra})^2 = 133.4 \text{ W}$$

11-11

$$R_{phase-phase} = 1.1\,\Omega$$

$$\therefore \quad R_s = \frac{R_{phase-phase}}{2} = 0.55\,\Omega$$

<u>No-Load Test:</u>

$$V_a = \frac{208}{\sqrt{3}} = 120\ V\,(rms)$$

$$I_a = 6.5A \quad , \quad P_{3-\phi} = 175W$$

$$\therefore \quad Q = \sqrt{(V_a I_a)^2 - \left(\frac{P_{3-\phi}}{3}\right)^2} = \sqrt{(120\times 6.5)^2 - \left(\frac{175}{3}\right)^2}$$

$$= 777.8\ VA = \omega L_m I_a^2$$

$$\therefore \quad L_m = \frac{777.8}{377\times 6.5^2} = 48.8\ mH$$

$$\Rightarrow X_m = 18.4\,\Omega$$

<u>Blocked-Rotor Test:</u>

$$V_a = \frac{53}{\sqrt{3}} = 30.6\ V\,(rms)$$

$$I_a = 18.2\ A$$

$$P_{blocked,3-\phi} = 900W$$

$$3(R_s + R_r')\,I_a^2 = P_{blocked-3\phi}$$

$$\therefore \quad R_s + R_r' = \frac{900}{3\times 18.2^2} = 0.906\,\Omega$$

$$\therefore \quad R_r' = 0.906 - R_s = 0.906 - 0.55 = 0.356\,\Omega$$

$$\frac{V_a}{I_a} = |Z| = \sqrt{(R_s + R_r')^2 + (X_{ls} + X_{lr}')^2}$$

$$\therefore \quad 0.906^2 + (X_{ls} + X_{lr}')^2 = \left(\frac{30.6}{18.2}\right)^2$$

or,

$$(X_{ls} + X_{lr}') \simeq 2.0\ \Omega$$

Using Eq. 11-47

$$X_{ls} \simeq \frac{2}{3} X_{lr}'$$

$$\left(\frac{2}{3}+1\right) X_{lr}' = 2.0$$

$$\therefore \quad X_{lr}' = 1.2\ \Omega \quad \Rightarrow L_{lr}' = 3.18\ \text{mH}$$

$$\text{and} \quad X_{ls} = 0.8\ \Omega \quad \Rightarrow L_{ls} = 2.12\ \text{mH}$$

Problem 11-12

In Fig 11.31, $\vec{V}_s$ and $\vec{B}_{ms}$ are at right angles and $\vec{V}_s$ is along q-axis since $\vec{B}_{ms} = \hat{B}_{ms} \angle -90°$.

Similarly $\vec{e}_r$ will be along the A-axis, if $\vec{B}_{ms}$ were to be $\hat{B}_{ms} \angle \theta - 90°$. But since $\vec{B}_{ms} = \hat{B}_{ms} \angle -90°$, $\vec{e}_r = \hat{E}_r \angle 0°$, as shown in Fig. 11-31.

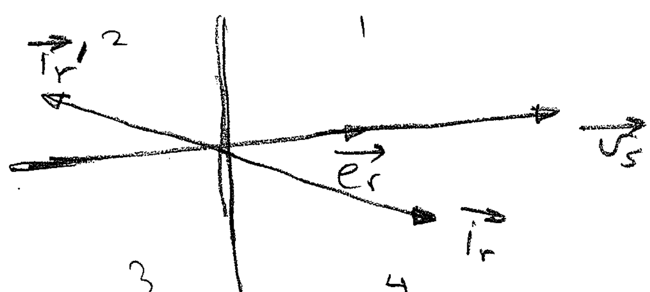
Problem 11-13

In the sub-synchronous mode, ω_{slip} and s are positive where

$$s = \frac{\omega_{slip}}{\omega_{slip}}.$$

The space vector diagram in Fig. 11.31 corresponds to a positive slip, as in this mode. The convention of $\vec{i}_r$ direction is as shown in Fig. 11.32.

Row 1:



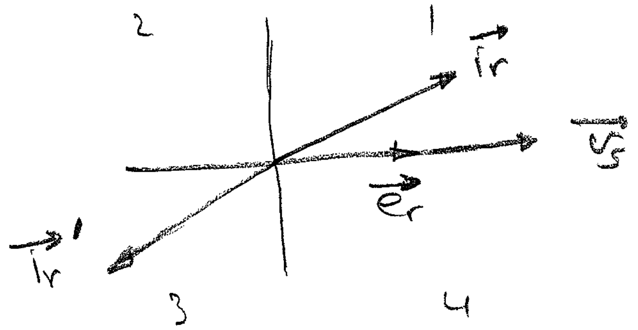
$P_r = +$

$Q_r = +$

$P_s = -$ Generating

$Q_r' = -$ producing reactive power & supplying it to the grid

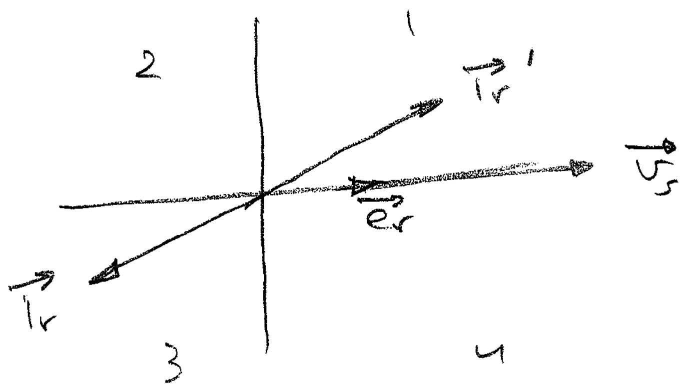
Row 2 :



$P_r = +$ | $P_s = -$ generating

$Q_r = -$ | $Q_r' = +$

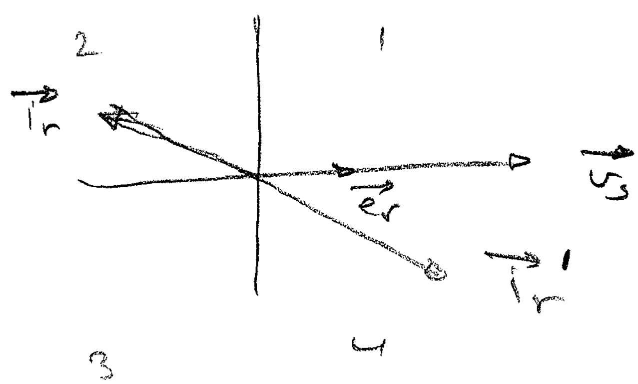
Row 3



$P_r = -$ | $P_s = +$ (motoring)

$Q_r = +$ | $Q_r' = -$

Row 4



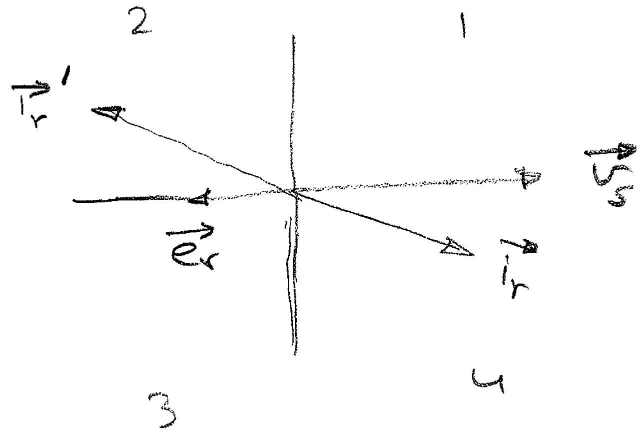
$P_r = -$ | $P_s = +$

$Q_r = -$ | $Q_r' = +$

Problem 11-14

In the supersync mode, S = negative

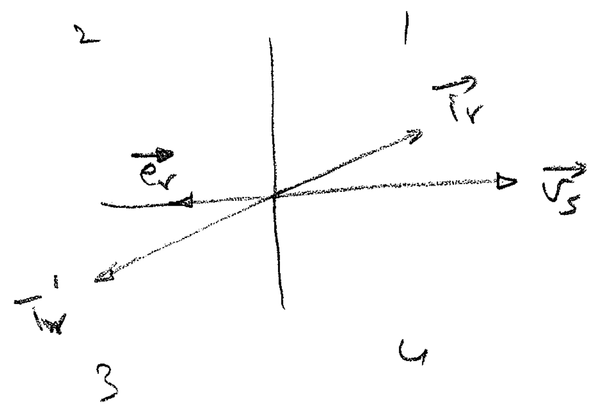
<u>Row 1</u>



$P_r = -$ $\quad$ $P_s = -$

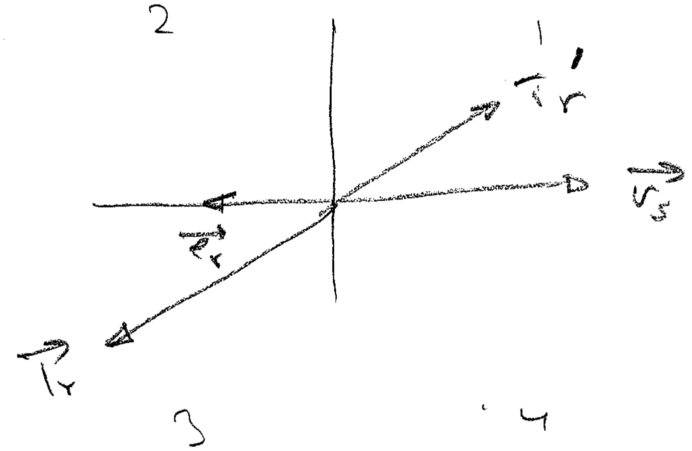
$Q_r = -$ $\quad$ $Q_r' = -$

<u>Row 2</u>



$P_r = -$	$P_s = -$
$Q_r = +$	$Q_r' = +$

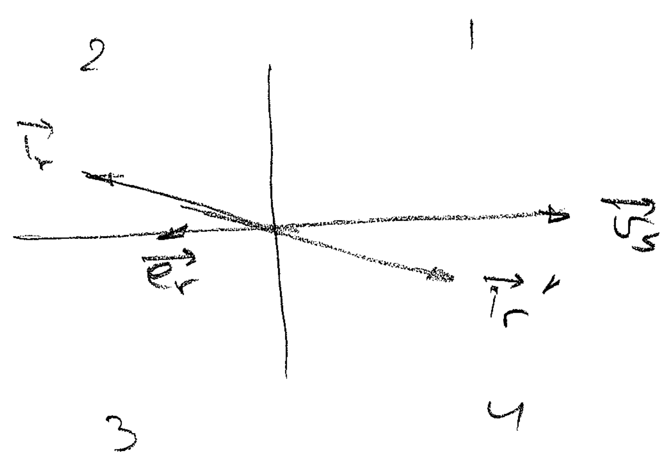
Row 3:



$P_r = +$ $\quad$ $P_s = +$

$Q_r = -$ $\quad$ $Q_r' = -$

Row 4



$P_r = +$ $\quad$ $P_s = +$

$Q_r = +$ $\quad$ $Q_r' = +$

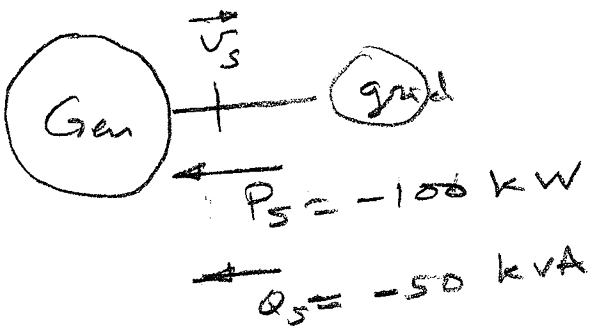
11-15

6 poles, $V_{LL,rms} = 480$ @ 60 Hz

$R_s = 0.008\,\Omega$, $X_{ls} = 0.1\,\Omega$, $X_m = 2.3\,\Omega$

$R_r' = 0.125\,\Omega$ and $X_{lr}' = 0.15\,\Omega$

$$\frac{n_s}{n_r} = \frac{1}{2.5}$$



With this information all the currents and voltages in the equivalent circuit can be calculated in order to calculate the desired quantities.

Chapter 12

12-1

for the centrifugal Load

$$T_L = C_n n_m^2$$

Such that $C_n = \dfrac{40 \text{ Nm}}{1746^2 \text{ rpm}^2} = 1.31\times10^{-5} \dfrac{\text{Nm}}{\text{rpm}^2}$

In steady state, similar to Eq. 12-9 for a centrifugal load,

$$k_{Tn}(n_{syn} - n_m) = C_n n_m^2$$

or,

$$C_n n_m^2 + k_{Tn} n_m - k_{Tn} n_{syn} = 0$$

$$\therefore \quad n_m = \frac{-k_{Tn} + \sqrt{k_{Tn}^2 + 4 C_n k_{Tn} n_{syn}}}{2 C_n}$$

for this motor, $k_{Tn} = 0.74$ NM/rpm.

~~There~~fore,

f (Hz)	n_{syn} (rpm)	n_m (rpm)	n_{slip} (rpm)
60	1800	1746	54
45	1350	1319.2	30.8
30	900	886.1	13.9
15	450	446.5	3.5

12-2 While supplying a centrifugal load

$$\frac{T_{em}}{T_{em,rated}} = \left(\frac{n_m}{n_{m,rated}}\right)^2$$

Therefore, Eq. 12-24 becomes

$$\hat{V}_a = 5.67f + R_s\left(\frac{n_m}{n_{m,rated}}\right)^2 \hat{I}'_{ra,rated}$$

where, $n_{m,rated} = 1746$ rpm and $\hat{I}'_{ra,rated} = 9.0\sqrt{2}$ A

$R_s = 1.5\,\Omega$

$$\therefore \quad \hat{V}_a = 5.67f + 19.1\left(\frac{n_m}{1746}\right)^2$$

f (Hz)	n_m (rpm) from Problem 12-1	$\hat{V}_a$ (Volts)
60 Hz	1746 rpm	359.3 V
45 Hz	1319.2 rpm	266.1 V
30 Hz	886.1 rpm	175.0 V
15 Hz	446.5 rpm	86.3 V

12-3 $T_{start} = T_{rated}$

$$\therefore \quad n_{syn,start} = n_{slip,rated} = 54 \text{ rpm}$$

$$\therefore \quad f_{start} = \frac{n_{syn,start}}{60}\,\frac{p}{2} = 1.8 \text{ Hz}$$

At the rated torque, $\hat{I}'_{ra,start} = \hat{I}'_{ra,rated} = \sqrt{2}\times 9.0$ A

From Eq. 12-20 and Example 12-2,

$$\hat{V}_{a,start} = 5.67 f_{start} + 1.5 \times \hat{I}_{ra,start}$$

$$= (5.67 \times 1.8) + 1.5 \times \sqrt{2} \times 9.0 = 29.3\text{V}$$

12-4

$$T_{braking} = T_{rated} \quad \therefore \quad \hat{I}'_{ra} = \hat{I}_{ra,rated} = \sqrt{2} \times 9.0\ A$$

$$n_m = 1746 \text{ rpm}$$

$$n_{slip} = -n_{slip,rated} = -54 \text{ rpm}$$

$$\therefore \quad n_{syn,braking} = n_m + n_{slip} = 1746 - 54$$
$$= 1692 \text{ rpm}$$

$$\therefore \quad f_{braking} = \frac{n_{syn,braking}}{60} \times \frac{p}{2} = 56.4 \text{ Hz}$$

and

$$\hat{V}_{a,braking} = 5.67\, f_{braking} + R_s(-\sqrt{2} \times 9.0)$$

due to the braking: current is 180° out-of-phase

$$= (5.67 \times 56.4) - 1.5 \times \sqrt{2} \times 9.0$$

$$= 300.7\ V$$

12.5

6-pole machine. At 60 Hz

$n_{syn} = 1200$ rpm

$s_{rated} = 1\%$

$\therefore \quad s_{rated} = \dfrac{n_{rated} - n_{syn}}{n_{syn}} = 0.01$

$\therefore \quad n_{rated} = 1.01 \times 1200 = 1212$ rpm

$\eta = \dfrac{P_{out}}{P_{in}} = 0.95$

$\therefore \quad P_{in,\,rated} = \dfrac{1.5 \text{ MW}}{0.95} = 1.579 \text{ MW}$

$T_{in,\,rated} = 1.579 \text{ MW} / 1212 \text{ rpm}$

C_p (= coefficient of performance) is to remain at its optimum value.

$\therefore \quad V_{wind,\,operating} = V_{wind,\,rated} \times \dfrac{n_{operating}}{n_{rated}}$

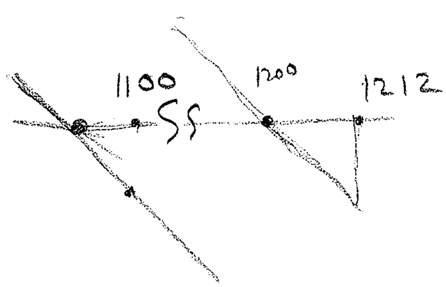
$= \dfrac{1100}{1212} \times V_{wind,\,rated}$

$P_{turbine} \propto \underset{(= \text{constant})}{C_p} \; V_{wind}^3$

$\therefore \quad P_{turbine} = P_{rated} \left(\dfrac{V_{wind,\,operating}}{V_{wind,\,rated}} \right)^3$

$= 1.5 \times \left(\dfrac{1100}{1212} \right)^3 = 1.12 \text{ MW}$

$= P_{in}$



$\therefore \quad T_{operating} = \dfrac{1.12 \text{ MW}}{1100 \text{ rpm}}$

$s_{operating} = s_{rated} \times \dfrac{T_{operating}}{T_{in,\,rated}} = 0.01 \times \dfrac{(1.12/1100)}{(1.579/1212)}$

$= 0.0078$

$$\frac{n_{op} - n_{syn}}{n_{syn}} = 0.0078$$

$$\therefore \quad n_{op} = 1.0078\, n_{syn} = 1100 \text{ rpm}$$

$$\therefore \quad n_{syn} = \frac{1100}{1.0078} = 1091.486 \text{ rpm}$$

$$\therefore \quad f = \frac{1091.486}{1200} \times 60 = 54.57 \text{ Hz}$$

and $V_{LL(rms)} = 600 \times \frac{54.57}{60} = 545.7\ V$

Chapter 13

13-1 Variable-Reluctance Step-Drive

Excitation sequence for a CCW rotation –

a – c – b – a

13-2 Permanent-Magnet Step Motor –

Excitation sequence for a CCW rotation –

$i_a^+, i_b^-, i_a^-, i_b^+, i_a^+, ---$

13-3 Hybrid Stepper Motor

$i_a^+, i_b^-, i_a^-, i_b^+, i_a^+, ---$

Chapter 14

14-1

$$1 \text{ year} = 365 \times 24 = 8760 \text{ hrs}$$

$$\therefore \tfrac{1}{2} \text{ yr} = 4380 \text{ hrs}$$

From Example 15-1

With the Standard motor,

$$\text{Annual Electricity Cost} = 28.09 \times 4380 \times 0.1$$

$$= \$12{,}303$$

With the premium-efficiency motor,

$$\text{Annual Electricity cost} = 27.17 \times 4380 \times 0.1$$

$$= \$11{,}900$$

$$\therefore \text{ Annual savings in operating cost} = 12{,}303 - 11{,}900$$

$$= \$403.$$

$$\therefore \text{ The payback period} = \frac{300}{403} \times 12 \approx 9 \text{ months}$$